Finite Difference Time Domain Modeling in Dispersive Media

Sarah Brown, Sherrette Yeates, & Carey Rappaport

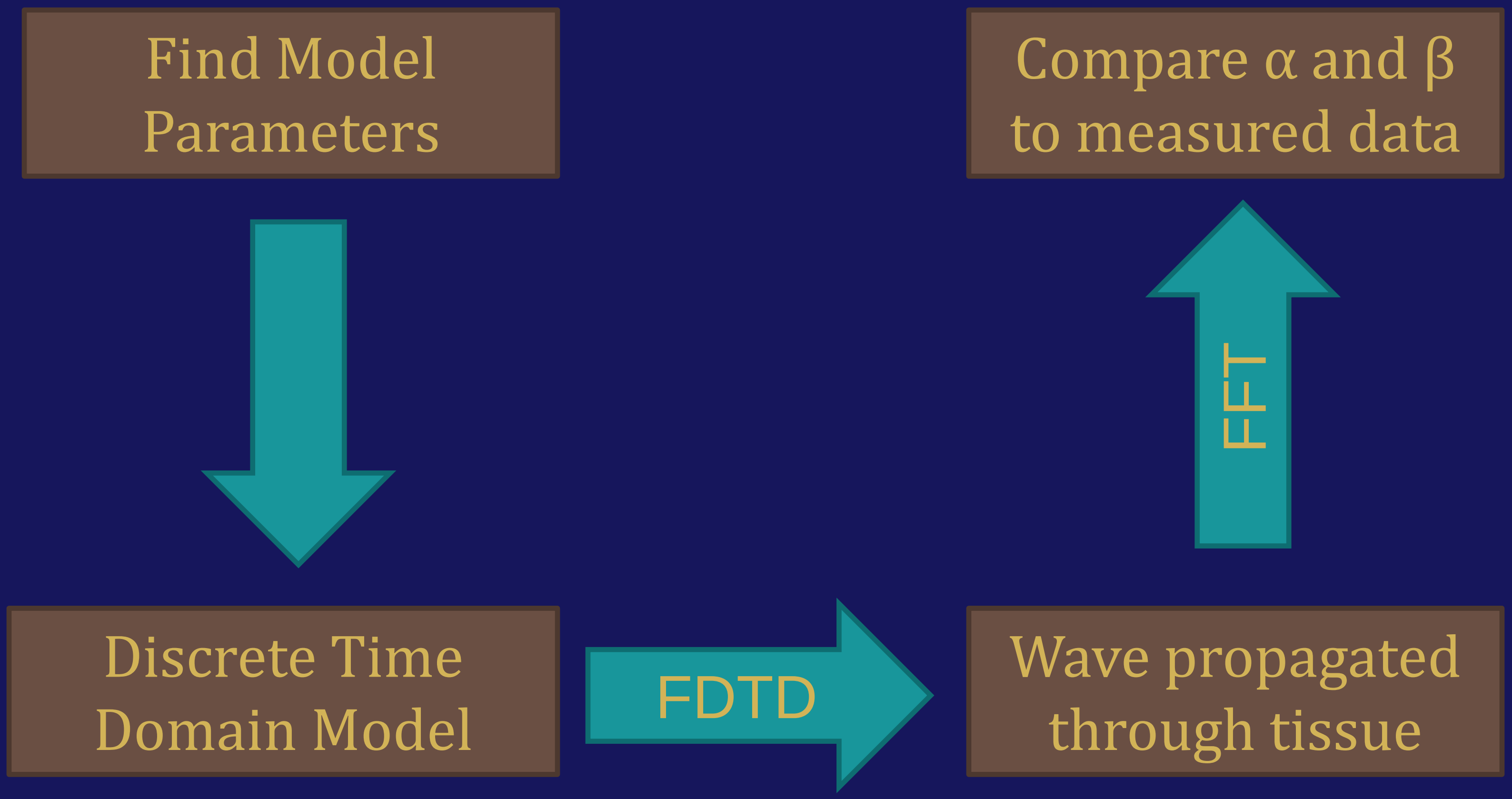
Gordon Center for Subsurface Sensing & Imaging Systems
Northeastern University, Boston, MA

PIERS Cambridge, MA, July 20010

Outline

- The Four Zero Conductivity uses less parameters than the traditional Cole-Cole expression
- Modeling can be applied to Medical Imaging System Design
- Model fits measured, published data
- Forward and Reverse FDTD Simulations show proper behavior

Developing the Forward Model



Four Zeros Conductivity Model

$$\sigma(Z) = \frac{b_0 + b_1 Z^{-1} + b_2 Z^{-2} + b_3 Z^{-3}}{1 + a_1 Z^{-1}} \qquad \varepsilon(Z) = \varepsilon_{av}$$

- Set Δt= 5ps
- Solve for b_0, b_1, b_2, and b_3, in terms of a_1
- Chose a_1 and Δz for stability
- Calculate k from data & model
- Compare α and β from model to data

Stability Analysis

Estimate Δz:

Courant's Stability condition

$$\Delta z_{min} = \frac{\sqrt{3}\, c\, \Delta t}{\sqrt{\varepsilon_1}}$$

Maintain 10points/λ

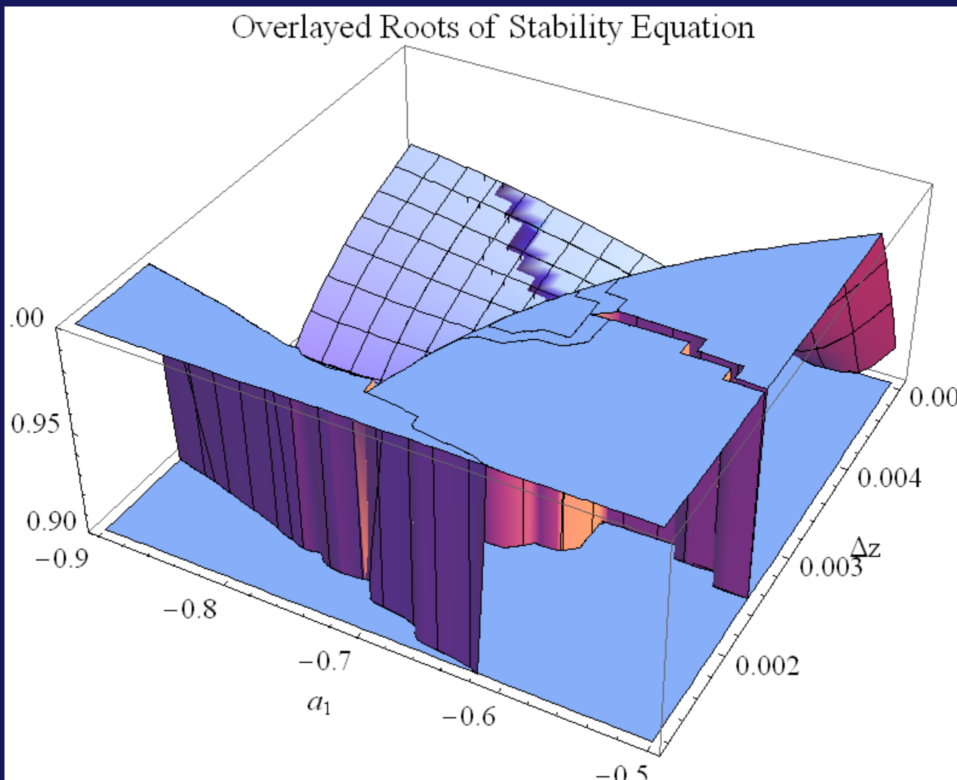
$$\Delta z_{max} = \frac{1}{10}\frac{c}{\sqrt{\varepsilon_2}\, f}$$

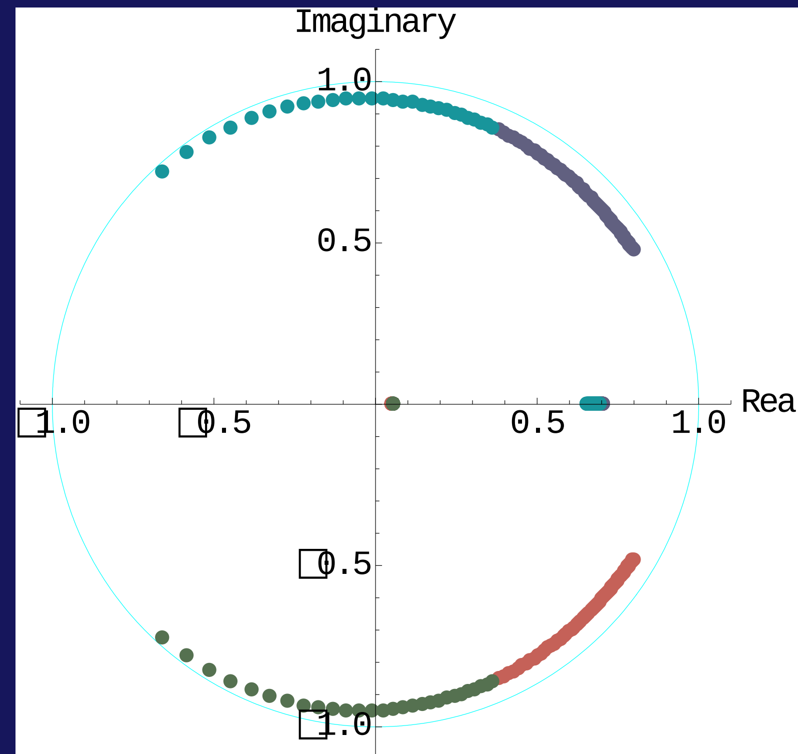
Von Neumann Stability Analysis

$$0 = 12r^2 + \varepsilon'(Z + 2 + Z^{-1}) + \frac{\sigma \Delta t}{\varepsilon_0}$$

$$r = \frac{c\Delta t}{\Delta z}$$

Forward Model Selecting a_1





$\Delta z_{min} = 0.0012m$

$\Delta z_{max} = 0.005m$,

$a_1 = [-.6, -.8]$

$\Delta z_{min} = 0.0012m$

$\Delta z_{max} = 0.005m$,

$a_1 = -.7$

Artificial Loss

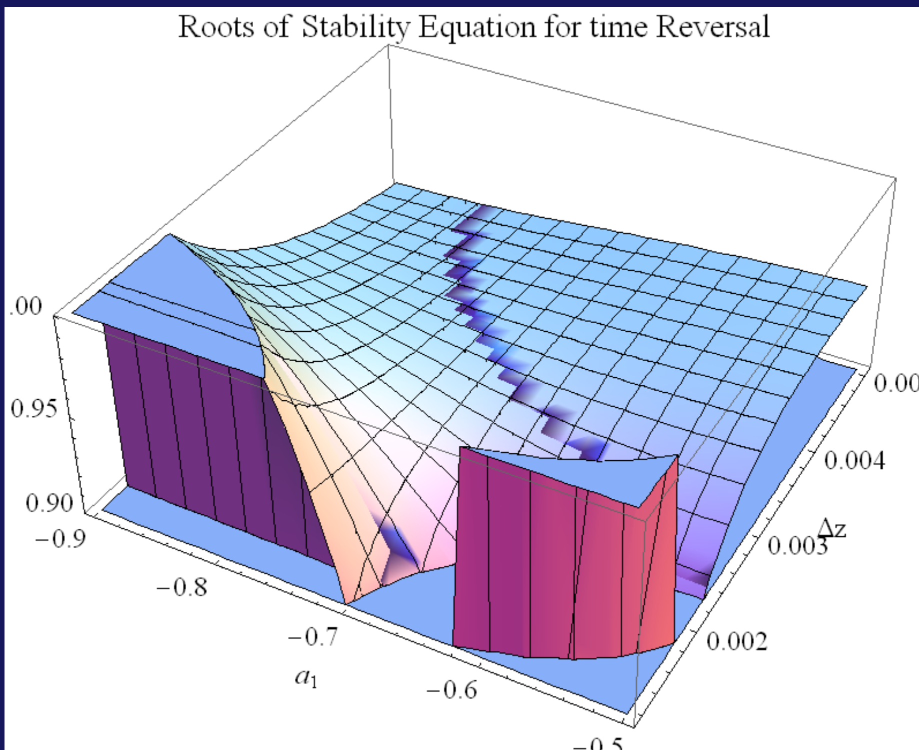
Add Artificial Loss, σ_0, for time reversal

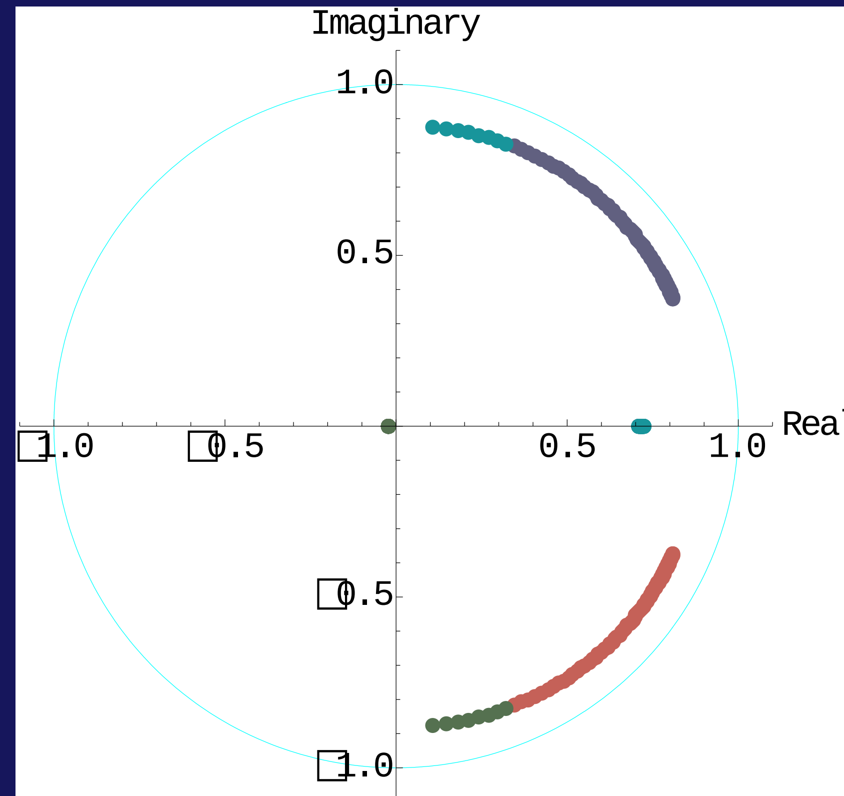
$$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E} + \frac{\sigma_0}{\epsilon'} \vec{E} \qquad \nabla \times \vec{E} = \mu_0 \frac{\partial \vec{H}}{\partial t} + \frac{\sigma_0}{\epsilon'} \vec{H}$$

Revised Stability condition:

$$0 = 12r^2 + \varepsilon'(Z + 2 + Z^{-1}) + \frac{\sigma(Z)\Delta t}{\varepsilon_0}(Z - 1)$$
$$+ \frac{\sigma(Z)\sigma_0 \Delta t}{\varepsilon' \varepsilon_0} + 2\frac{\sigma_0 \Delta t}{\varepsilon_0}(1 - Z^{-1})$$
$$+ \frac{{\sigma_0}^2 \Delta t^2}{\varepsilon' {\varepsilon_0}^2} Z^{-1}$$

Time Reversed Model: Selecting Artificial Loss





Δz_{min} = 0.0012m , Δz_{max} = 0.005m

a_1 = [-.6, -.8], σ_0 = 5

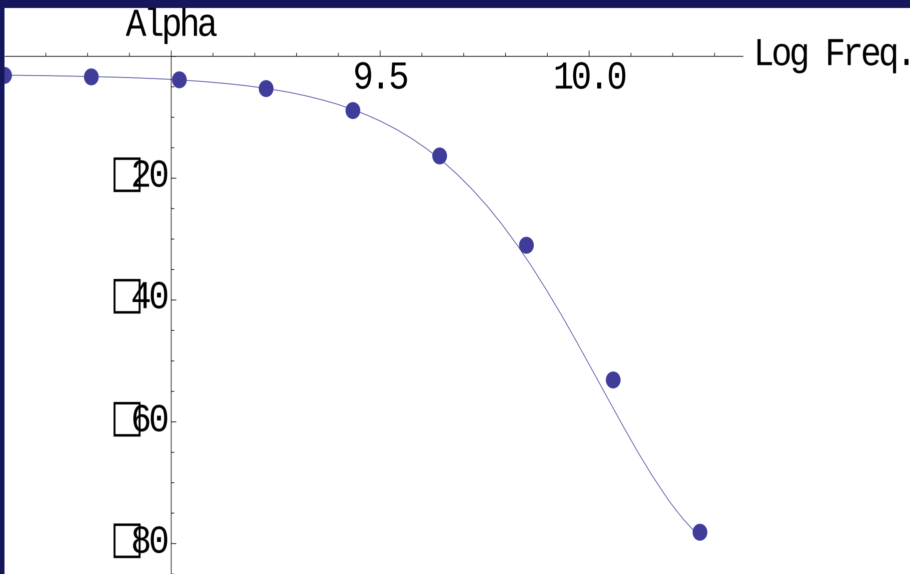
Δz_{min} = 0.0012m,

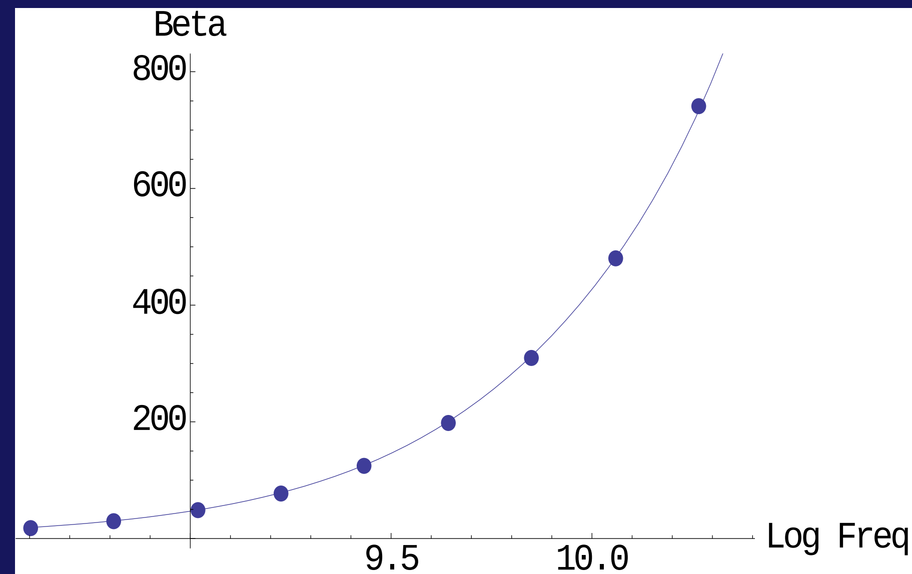
Δz_{max} = 0.005m,

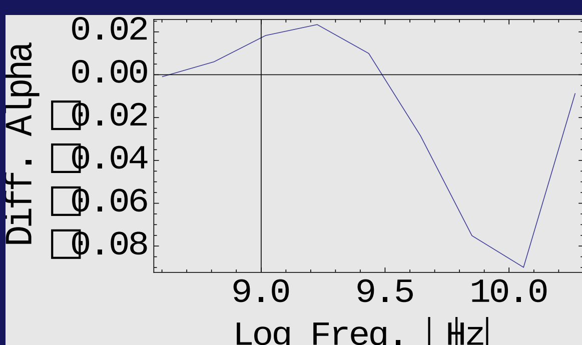
a_1 = .7, σ_0 = 5

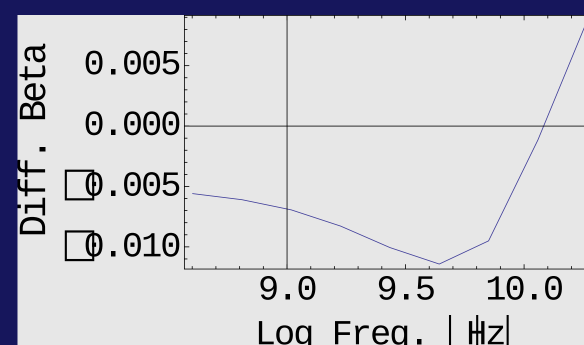
Verify Model

$$k(\omega) = \frac{\omega}{c_0 \varepsilon_0}\sqrt{\varepsilon_{av} - j\frac{\sigma(\omega)}{\omega}} = \beta + j\alpha$$



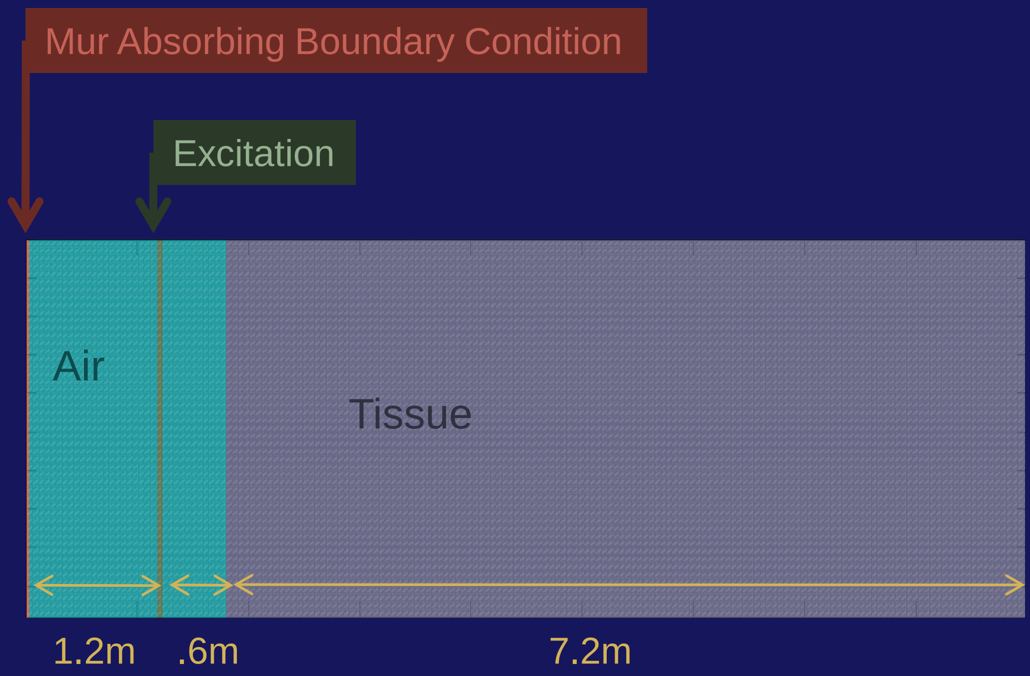


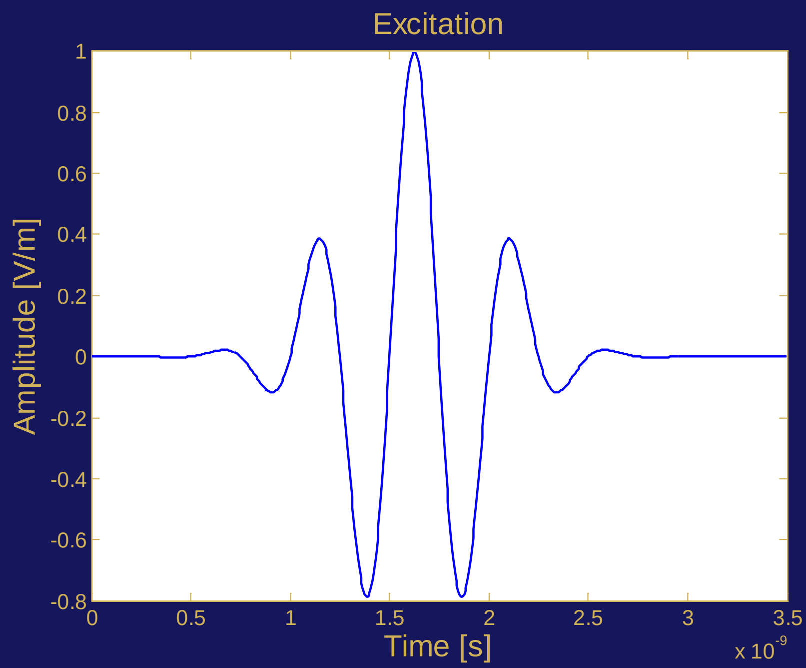




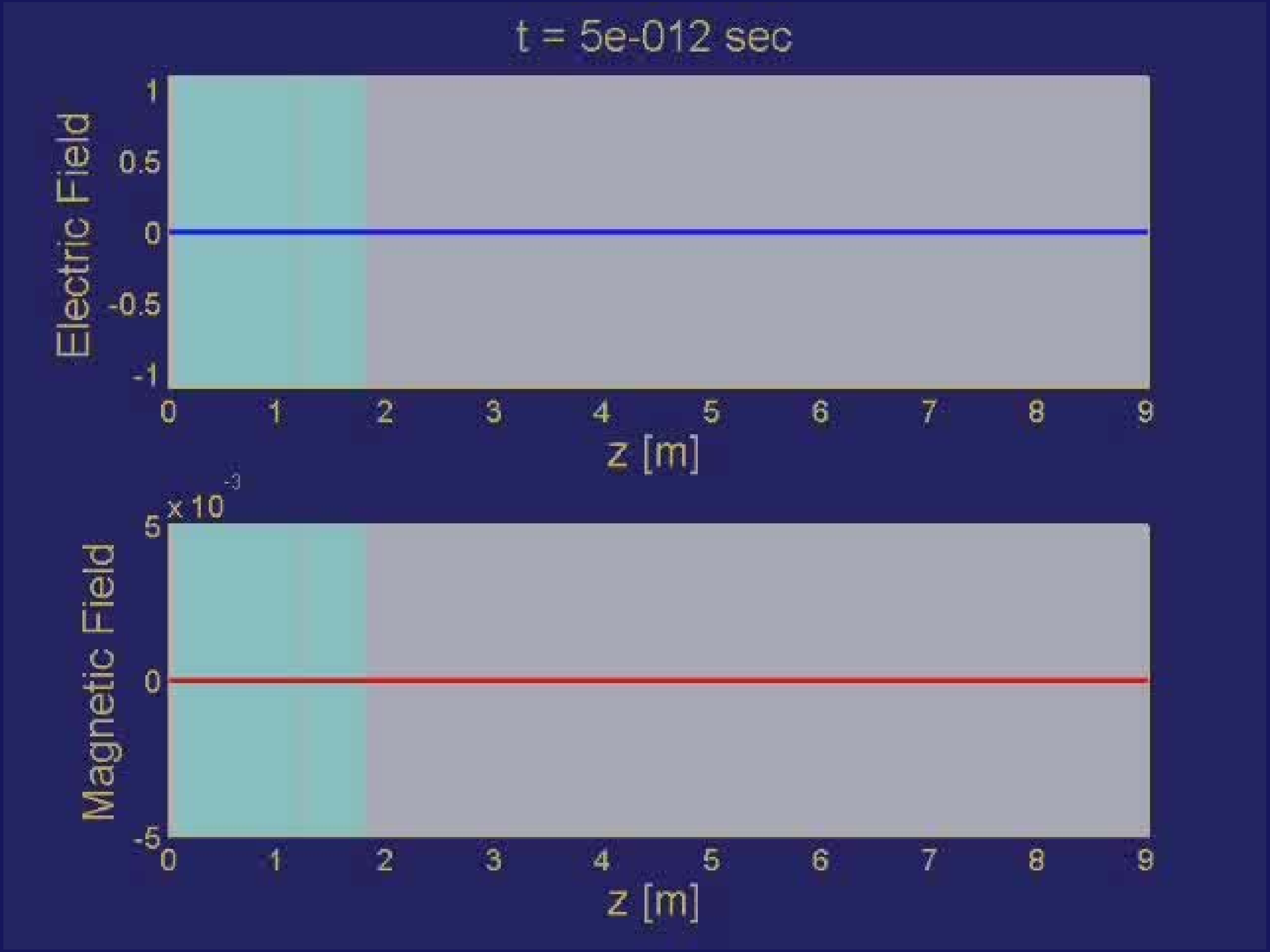
1- D FDTD

$$\nabla\times\left(H^{n-\frac{1}{2}}+a_1H^{n-\frac{3}{2}}\right)=\left(\frac{\varepsilon_{av}}{\Delta t}+b_0\right)E^n+\left((a_1-1)\frac{\varepsilon_{av}}{\Delta t}+b_1\right)E^{n-1}+\left(-a_1\frac{\varepsilon_{av}}{\Delta t}+b_2\right)E^{n-2}+b_3E^{n-3}$$





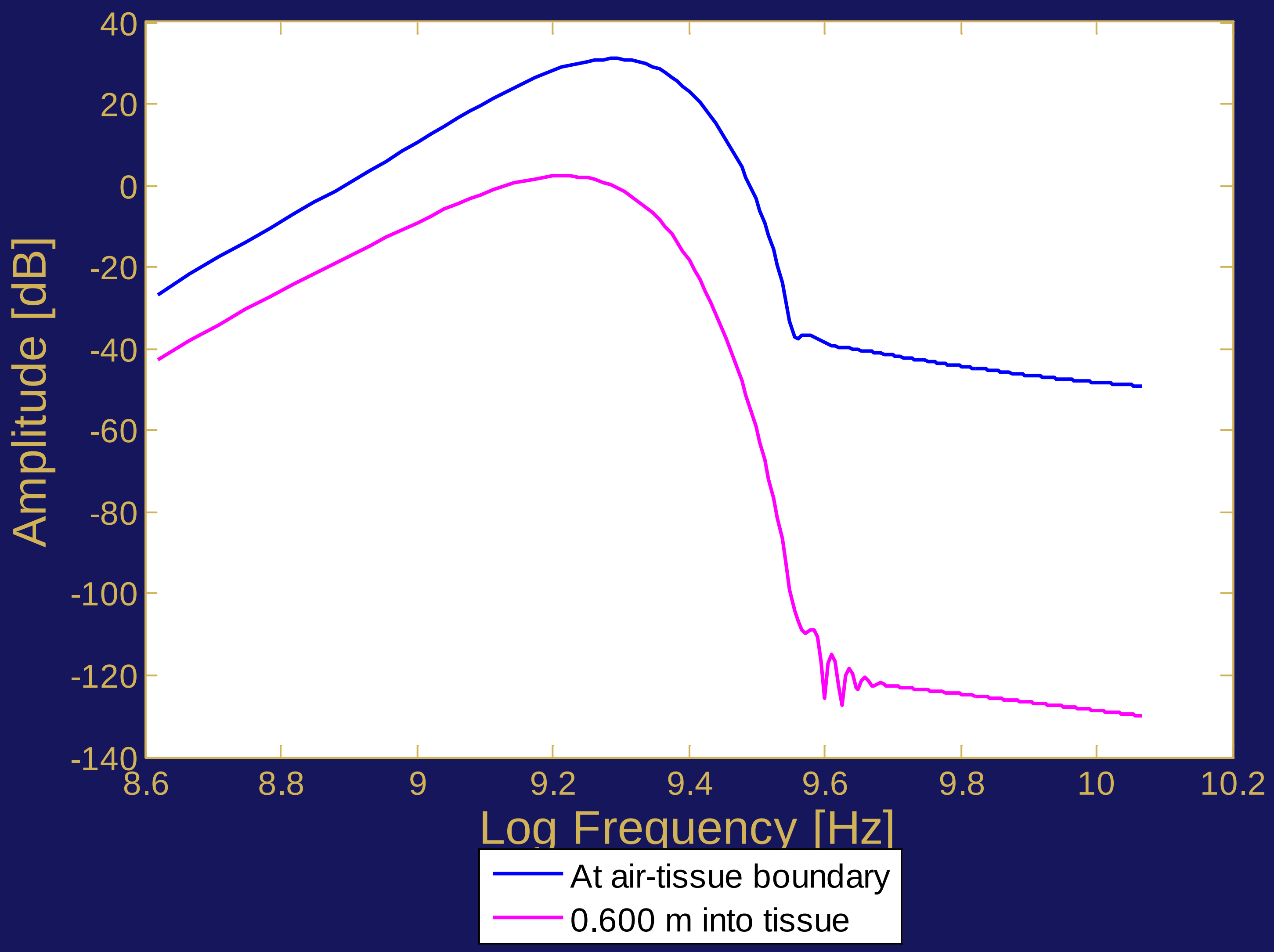
Forward Simulation



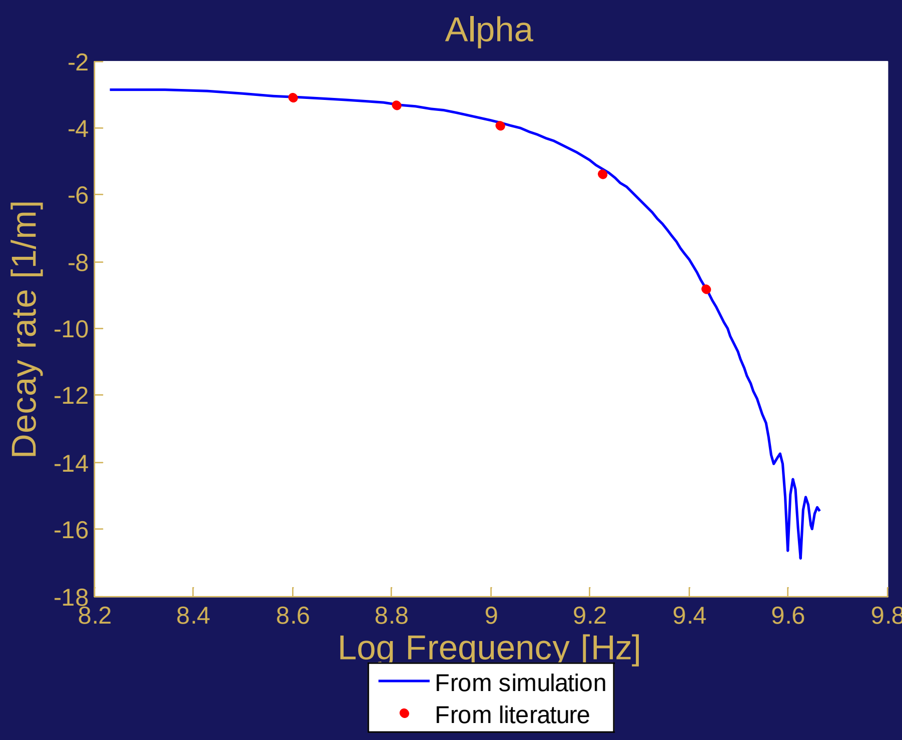
Verification of Simulation

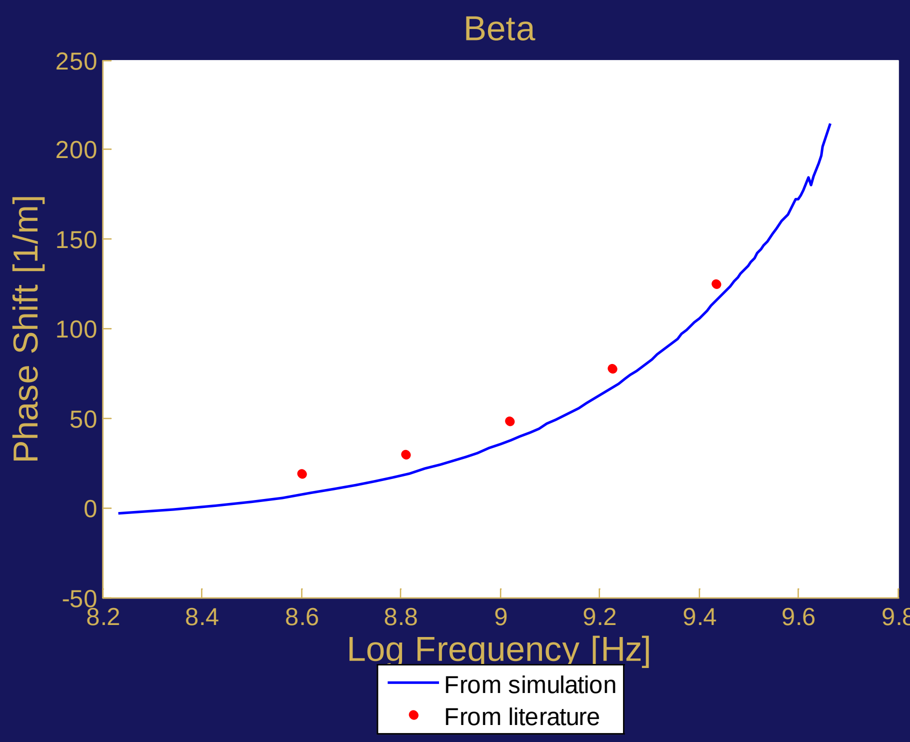
- FFT Ex Field at air-tissue boundary
- FFT Ex Field at 60cm into tissue, center of pulse at 9ns
- Divide, take log, separate α and β

Verification of Forward Simulation



Simulation v. Data



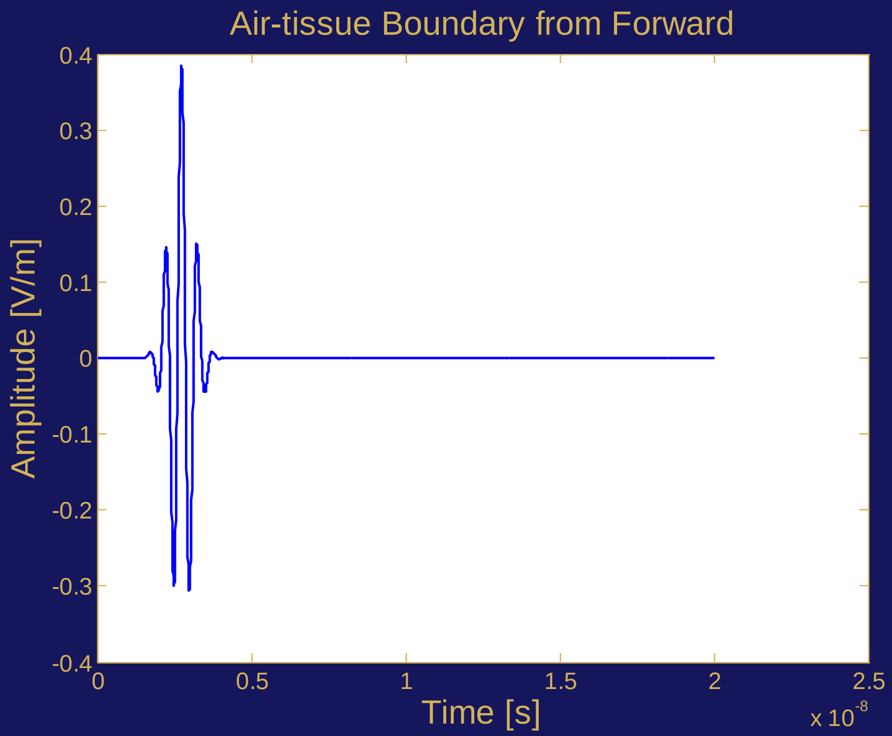


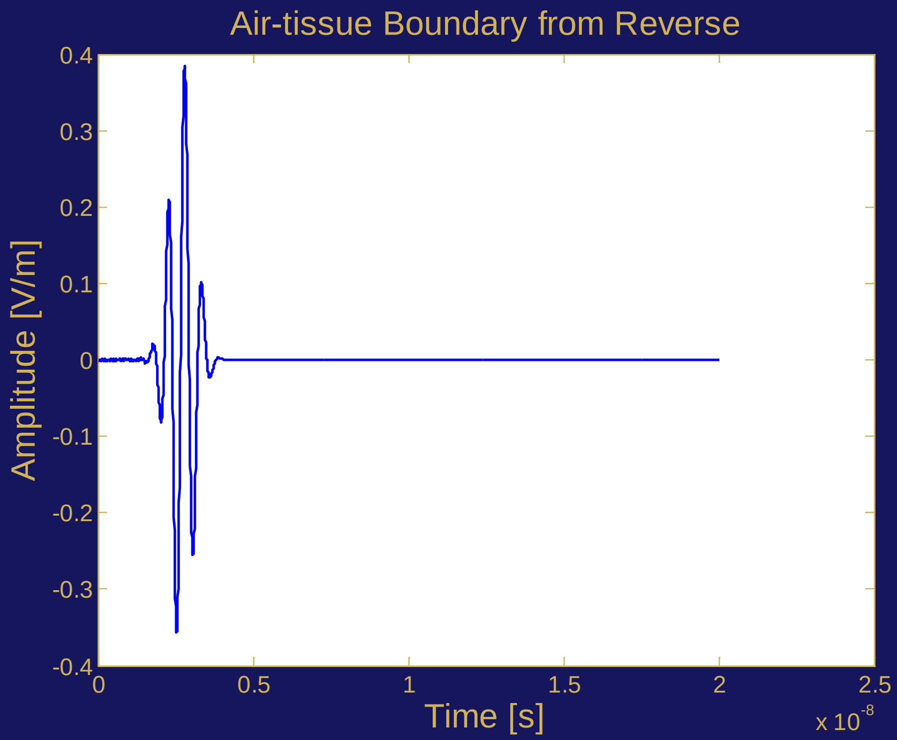
Time Reversal

- Excite with recording from z = 2.4m
- Apply artificial loss at each step
- Remove artificial loss at air-tissue boundary
- Compare with forward

$$E_x(z = 2.4m) = E_{xr}(z = 2.4m) * e^{\frac{n\Delta t}{\tau}}$$

Verification of Time Reversal





Conclusions

- Four Zero model is stable for biological tissue
- Forward FDTD agrees with frequency domain analysis of tissue properties
- Time Reversed FDTD with artificial loss for gain removal is stable

Acknowledgements: Gordon-CenSSIS (ERC Award Number EEC-9986821)

the New England Louis Stokes Alliance for Minority Participation

References

1. C. Rappaport and S. Winton, “Modeling dispersive soil for FDTD computation by fitting conductivity parameters,” in *12th Annu. Rev. Progress Applied Computational Electromagnetic Symp. Dig.*, Mar. 1997, pp. 112–118.

2. W. Weedon, and Rappaport, C., “A General Method for FDTD Modeling of Wave Propagation in Arbitrary Frequency-Dependent Media,” *IEEE T. Ant. Prop.*, vol. 45, Mar. 1997, pp. 401-410.

3. C. Rappaport, E. Bishop and P. Kosmas “Modeling FDTD wave propagation in dispersive biological tissue using a single pole Z-transform function”, *International Conference of the IEEE Eng. in Medicine & Biology Society*, Cancun, Mexico, September 2003.

4. M. Jalalinia, Rappaport, C., “Propagation and Stabilization in Dispersive Media,” *Progress in Electromagnetics Research Symposium*, Cambridge, MA, July 2008, p. 62.