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Robust Unscented Kalman Filter for Power System Dynamic State Estimation using PMUs

Lamine Mili
Professor, IEEE Life Fellow, lmili@vt.edu
Junbo Zhao
Assistant Professor, zjunbo@vt.edu
Bradley Department of Electrical and Computer Engineering
Virginia Tech, Northern Virginia Center
Falls Church, VA 22043, USA

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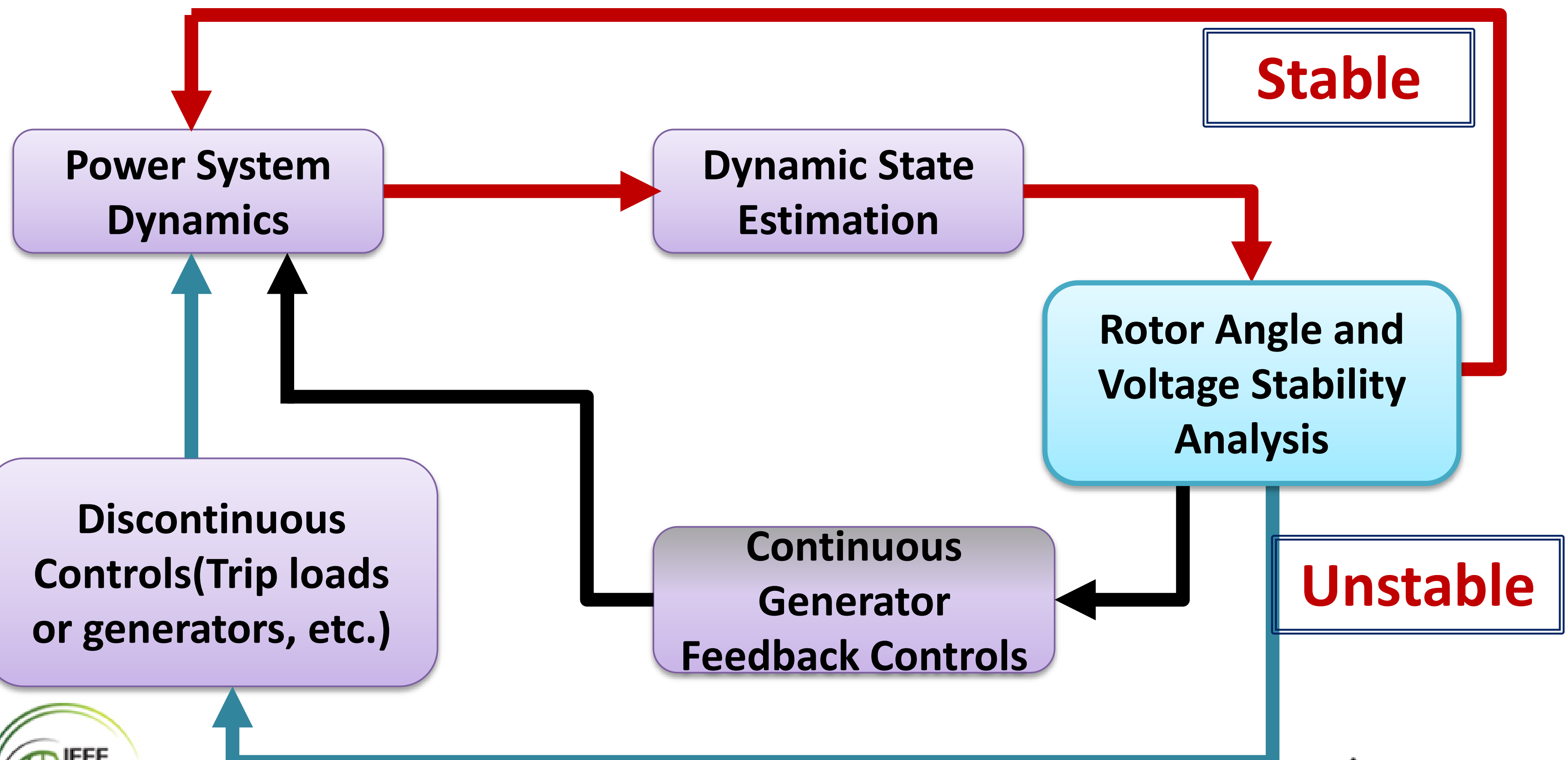
Dynamic State Estimation- Motivations

➢ **<u>Dynamic Security Analysis</u>:** Perform real-time state prediction and filtering for rotor angle and voltage stability.

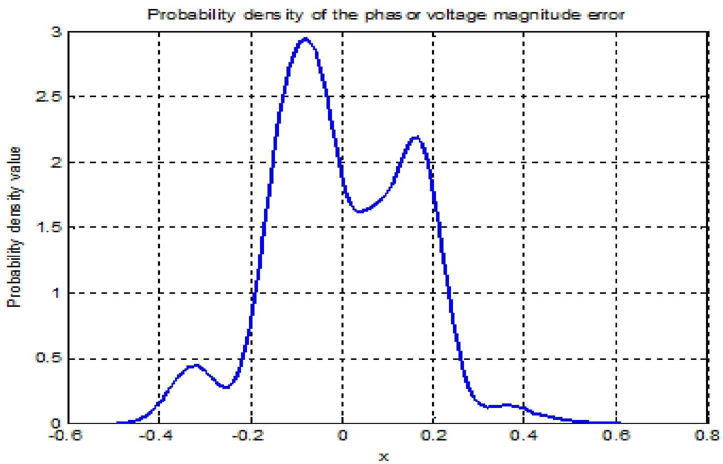


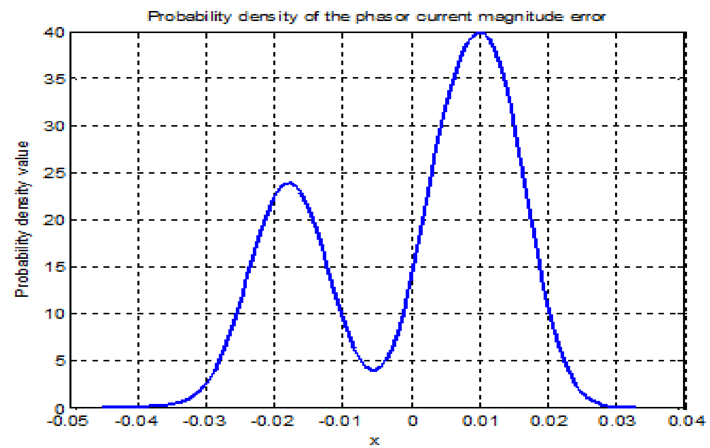
Why Robust DSE for Power Systems?

PMU data quality issues:

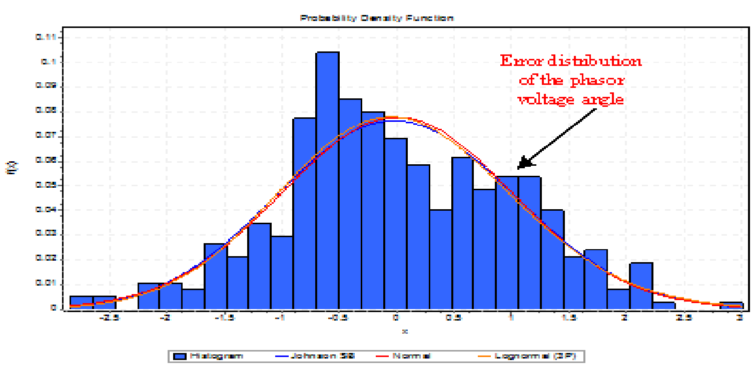
- Large measurement bias
- Non-Gaussian noise
- Data dropout/packet loss
- Loss of GPS synchronization
- Measurement delays
- Bad/missing timestamps
- Cyber attack…

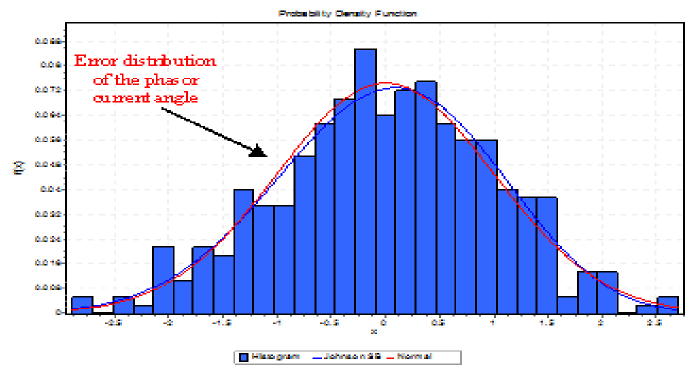
Non-Gaussian PMU noise (H. Huang from PNNL)



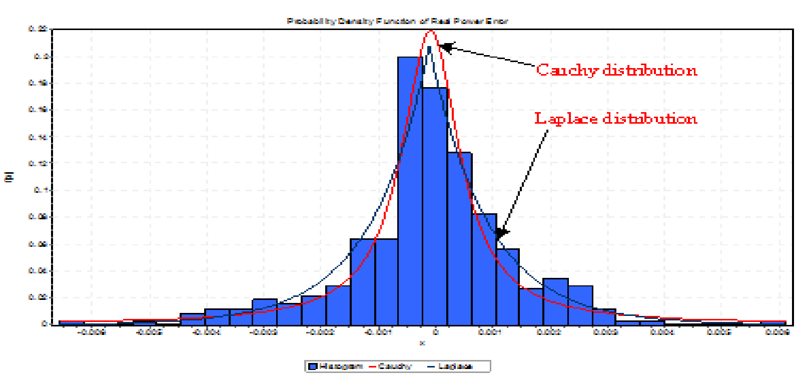


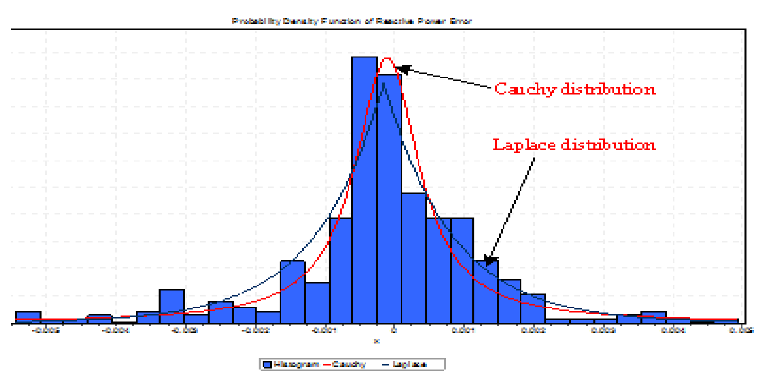
Voltage and current magnitudes





Voltage and current angles





P and Q

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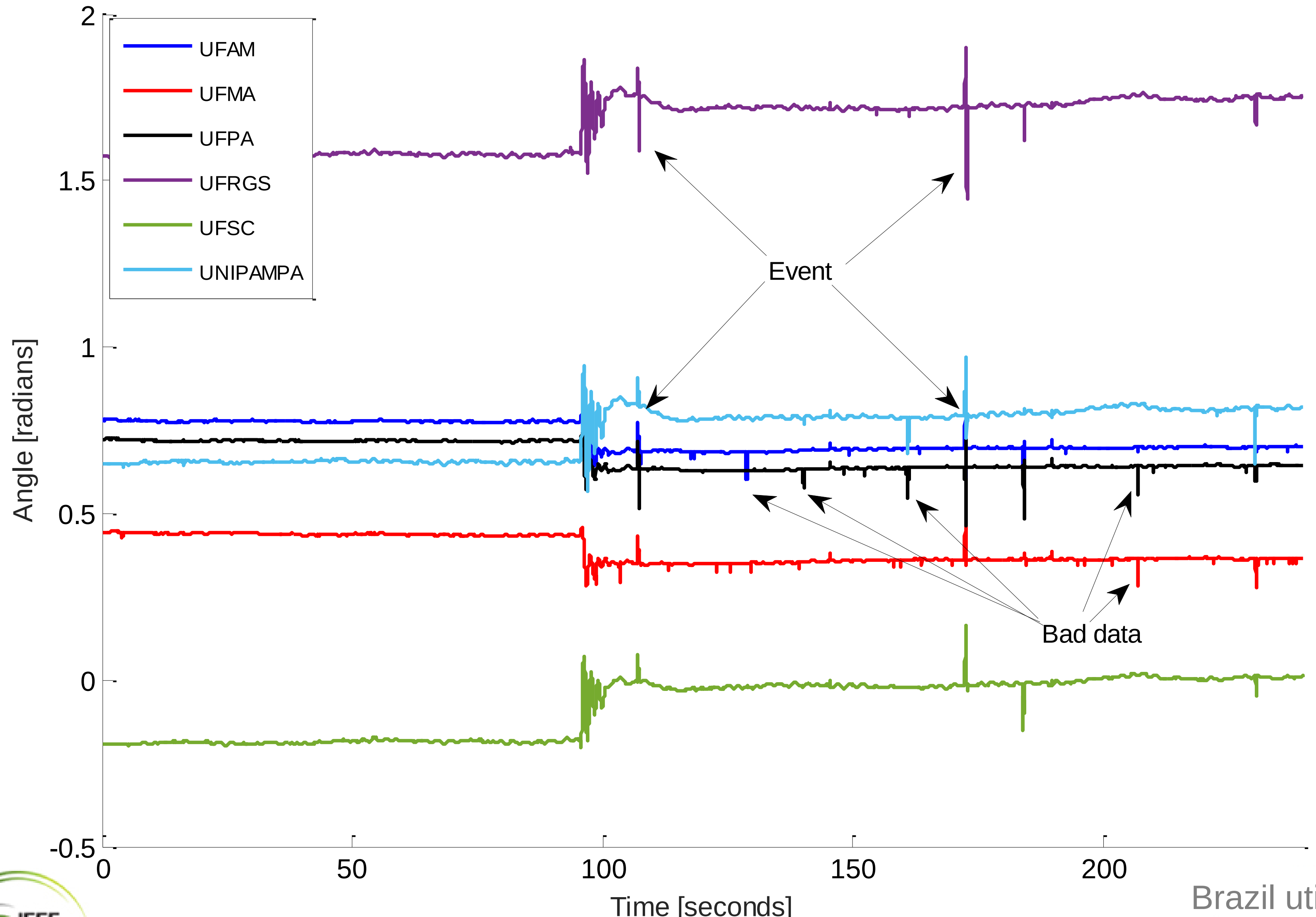
Occurrence of Outliers (PMU Data from Brazil Utility)



Brazil utility

How Does the KF Work?

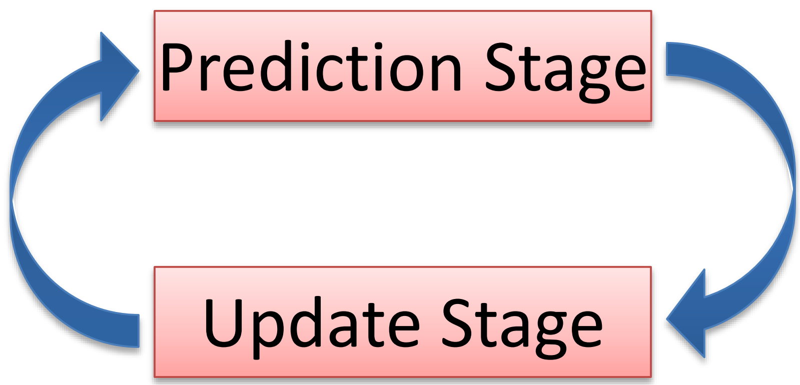
Predict the state

$$\underline{\hat{x}}_{k|k-1} = \underline{F}_{k-1}\underline{\hat{x}}_{k-1|k-1}$$

Compute covariance

$$\underline{\Sigma}_{k|k-1} = \underline{F}_{k-1}\underline{\Sigma}_{k-1|k-1}\underline{F}_{k-1}^{T} + \underline{W}_{k}$$

System process noise covariance matrix



Observation noise covariance matrix

Compute the filter gain

$$\underline{K}_{k} = \underline{\Sigma}_{k|k-1}\underline{H}_{k}^{T}[\underline{H}_{k}\underline{\Sigma}_{k|k-1}\underline{H}_{k}^{T} + \underline{R}_{k}]^{-1}$$

Correct the prediction

$$\underline{\hat{x}}_{k|k} = \underline{\hat{x}}_{k|k-1} + \underline{K}_{k}[\underline{z}_{k} - \underline{H}_{k}\underline{\hat{x}}_{k|k-1}]$$

Update the covariance

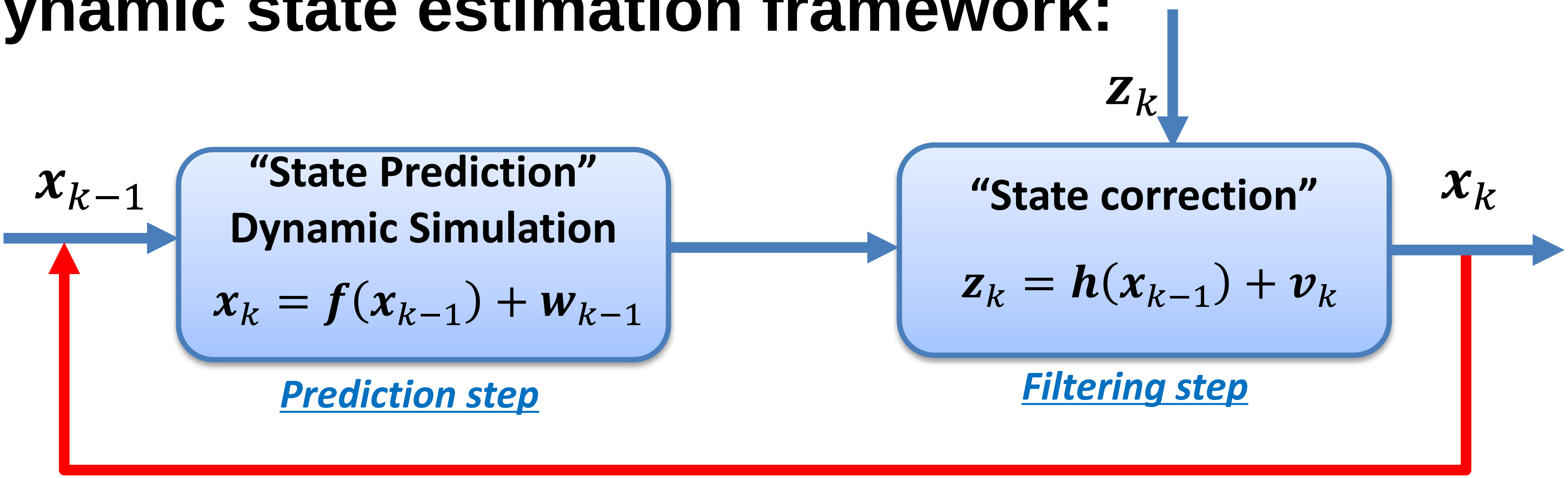
$$\underline{\Sigma}_{k|k} = (\underline{I} - \underline{K}_{k}\underline{H}_{k})\underline{\Sigma}_{k|k-1}$$

Problem Formulation

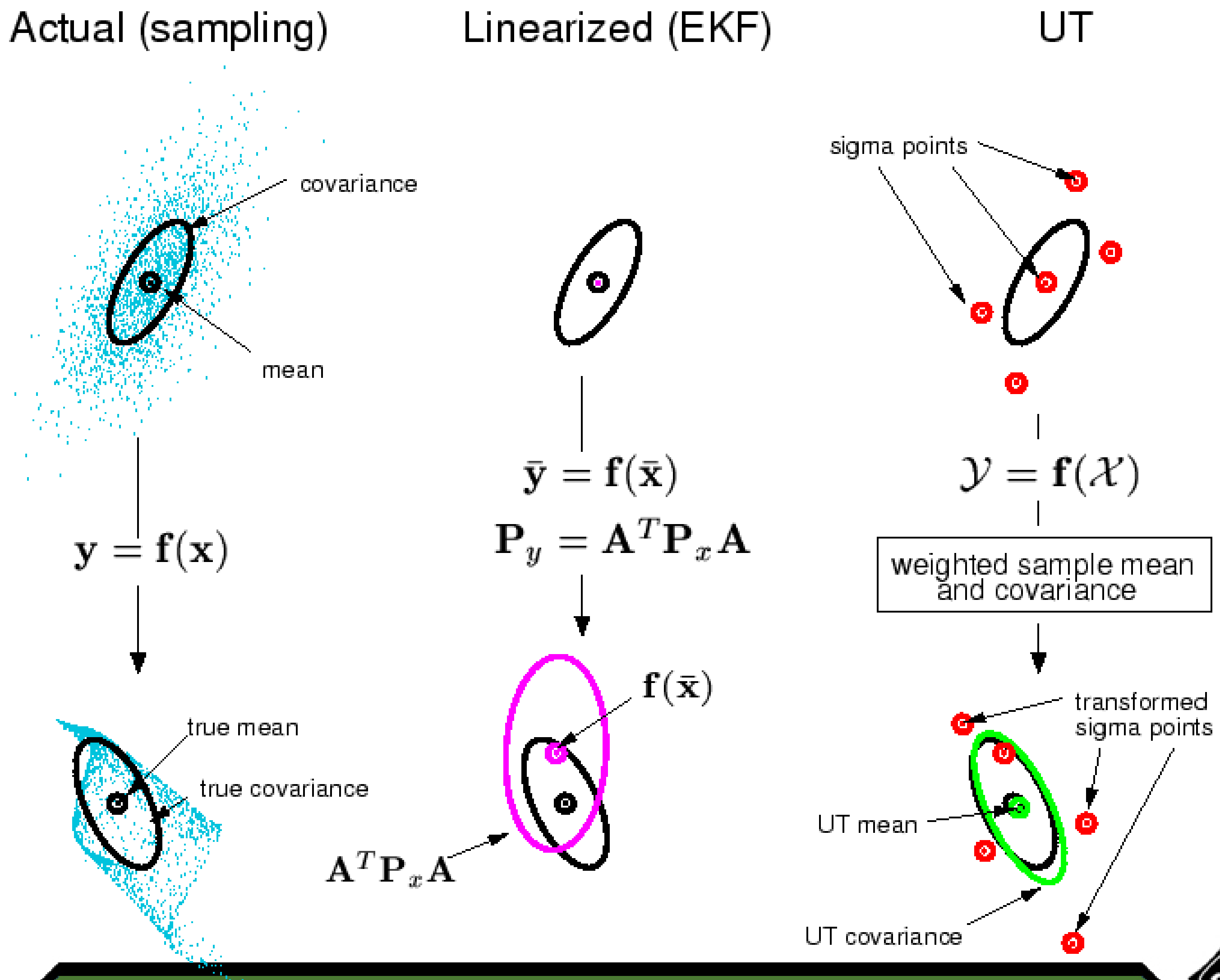
Dynamical system discrete-time state-space model:

$$\boldsymbol{x}_k = \boldsymbol{f}(\boldsymbol{x}_{k-1}) + \boldsymbol{w}_{k-1} \qquad \mathbb{E}[\boldsymbol{w}_{k-1}\boldsymbol{w}_{k-1}{}^T] = \boldsymbol{Q}_k$$
$$\boldsymbol{z}_k = \boldsymbol{h}(\boldsymbol{x}_{k-1}) + \boldsymbol{v}_k \qquad \mathbb{E}[\boldsymbol{v}_k\boldsymbol{v}_k{}^T] = \boldsymbol{R}_k \tag{1}$$

Dynamic state estimation framework:



Extended Kalman filter (EKF) and Unscented Kalman filter (UKF)



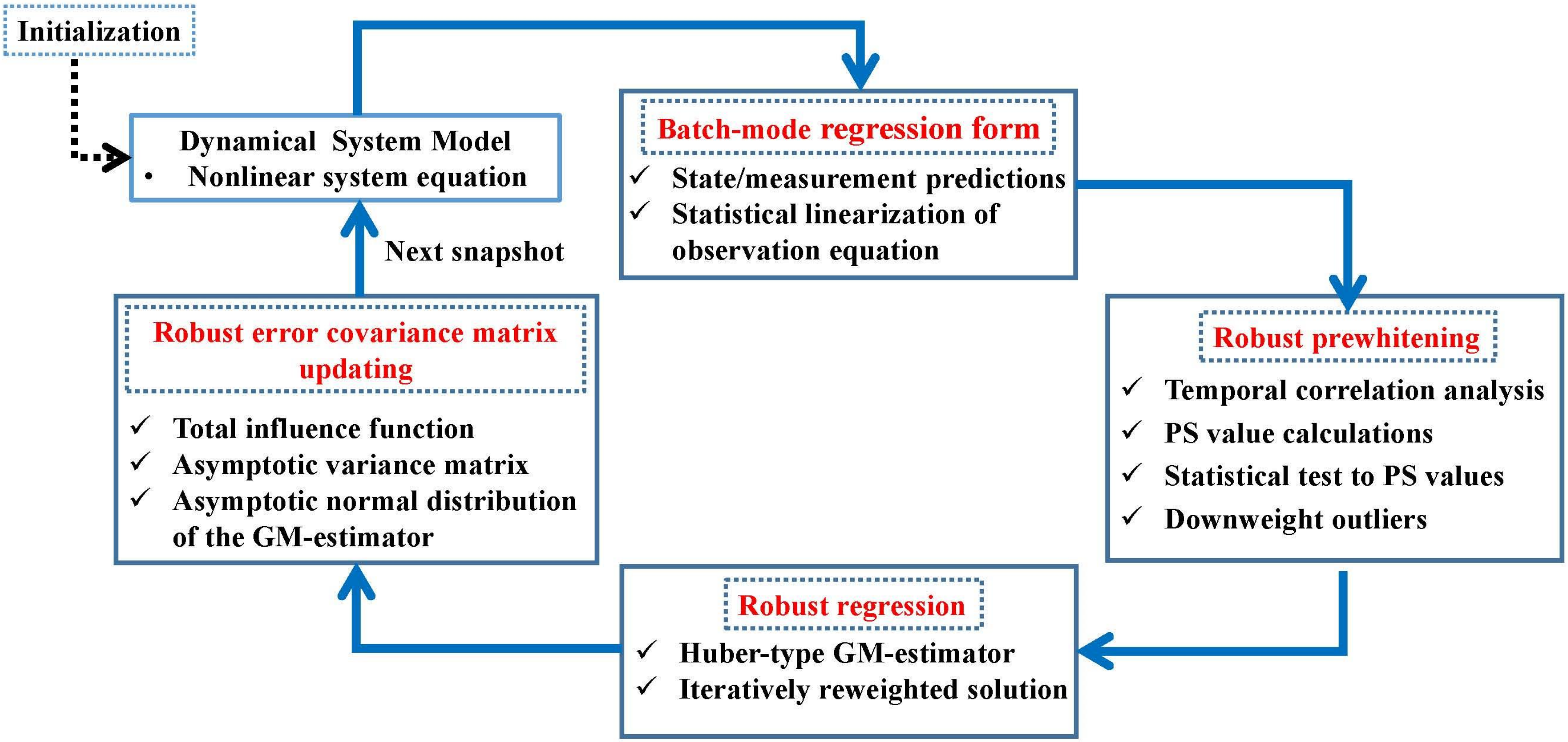
Drawbacks of EKF and UKF

- They are based on **Least Squares Estimator** and thus highly sensitive to deviations from the assumptions;
- They provide biased results with **non-Gaussian noise**;
- Lack of robustness to any type of outliers: **observation, innovation and structural outliers**;
- Sensitive to **cyber attacks, measurement losses, etc.**

Table I Definitions of the three types of outliers

Outlier type	Types of noise	Affected components
Observation outlier	Observation noise, $\boldsymbol{v}_k$	$\boldsymbol{z}_k$
Innovation outlier	System process noise, $\boldsymbol{w}_{k-1}$	$\hat{\boldsymbol{x}}_{k\|k-1}$
Structural outlier	Structural errors in $\boldsymbol{f}(\boldsymbol{x}_{k-1})$, $\boldsymbol{h}(\boldsymbol{x}_k)$	$\boldsymbol{z}_k$, $\hat{\boldsymbol{x}}_{k\|k-1}$, $\boldsymbol{\Sigma}_{k\|k-1}$, $\boldsymbol{\Sigma}_{k\|k}$

Proposed Robust GM-UKF



Batch-Mode Regression Form

Combine the measurement equation and the prediction equation to obtain the **batch-mode regression form** :

Prediction equation: Predicted state Prediction error

True value

$$\boldsymbol{x}_k = \widehat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{\zeta}_{k|k-1} \quad \textbf{(2)}$$

Measurement equation: Perform statistical linearization of $\boldsymbol{h}(\boldsymbol{x}_k)$ around the predicted state $\widehat{\boldsymbol{x}}_{k|k-1}$:

$$\boldsymbol{z}_k = \boldsymbol{h}(\widehat{\boldsymbol{x}}_{k|k-1}) + \boldsymbol{H}_k(\boldsymbol{x}_k - \widehat{\boldsymbol{x}}_{k|k-1}) + \boldsymbol{v}_k + \boldsymbol{e}_k \quad \textbf{(3)}$$

where $\boldsymbol{H}_k = \left(\boldsymbol{P}_{k|k-1}^{xz}\right)^T \left(\boldsymbol{P}_{k|k-1}^{xx}\right)^{-1}$ and $\boldsymbol{e}_k$ is the statistical linearization error.

Batch-mode regression form

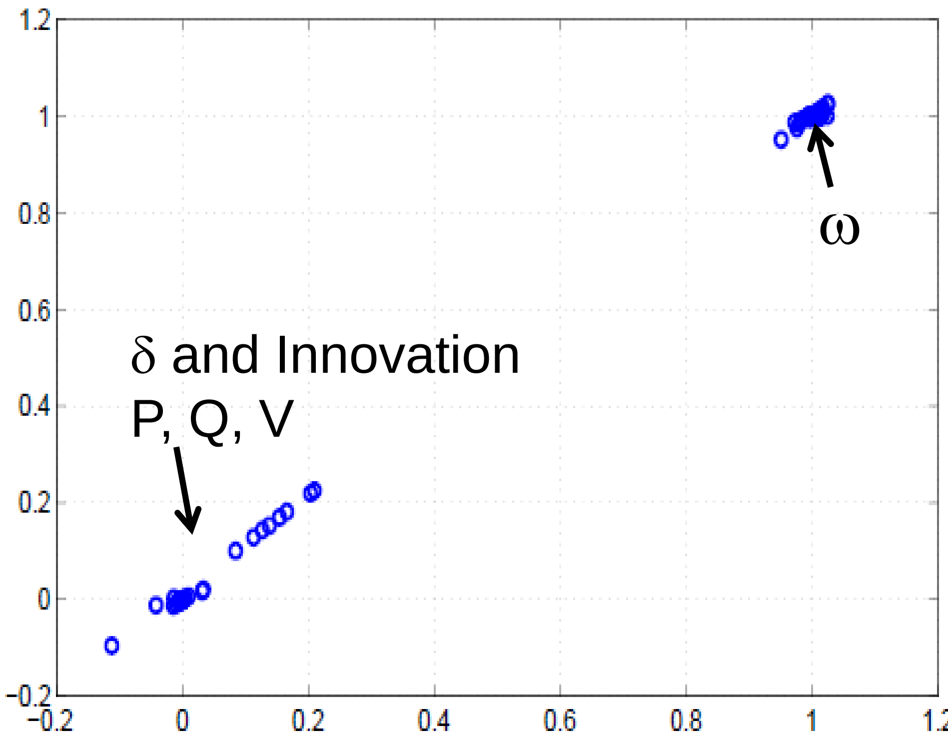
$$\underbrace{\begin{bmatrix} \boldsymbol{z}_k - \boldsymbol{h}(\widehat{\boldsymbol{x}}_{k|k-1}) + \boldsymbol{H}_k \widehat{\boldsymbol{x}}_{k|k-1} \\ \widehat{\boldsymbol{x}}_{k|k-1} \end{bmatrix}}_{\tilde{\boldsymbol{z}}_k} = \underbrace{\begin{bmatrix} \boldsymbol{H}_k \\ \boldsymbol{I} \end{bmatrix}}_{\widetilde{\boldsymbol{H}}_k} \boldsymbol{x}_k + \underbrace{\begin{bmatrix} \boldsymbol{v}_k + \boldsymbol{e}_k \\ -\boldsymbol{\zeta}_{k|k-1} \end{bmatrix}}_{\tilde{\boldsymbol{e}}_k} \quad (4)$$

- The covariance matrix of the error $\tilde{\boldsymbol{e}}_k$ is given by

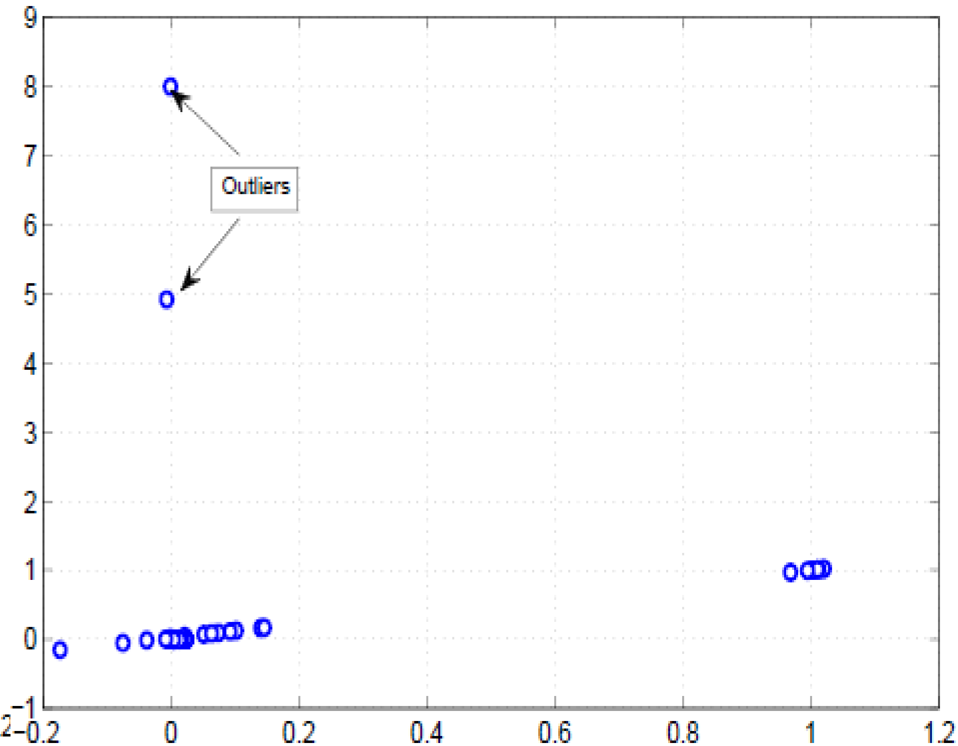
$$\mathbb{E}[\tilde{\boldsymbol{e}}_k \tilde{\boldsymbol{e}}_k{}^T] = \begin{bmatrix} \boldsymbol{R}_k + \widetilde{\boldsymbol{R}}_k & 0 \\ 0 & \boldsymbol{\Sigma}_{k|k-1} \end{bmatrix} = \boldsymbol{S}_k \boldsymbol{S}_k{}^T, \quad (5)$$

where $\mathbb{E}[\boldsymbol{e}_k \boldsymbol{e}_k{}^T] = \widetilde{\boldsymbol{R}}_k$ and $\boldsymbol{S}_k$ is obtained from **Cholesky decomposition** and used for **prewhitening** after outlier detection.

Detecting Outliers by PS



Scatter of matrix Z without outliers



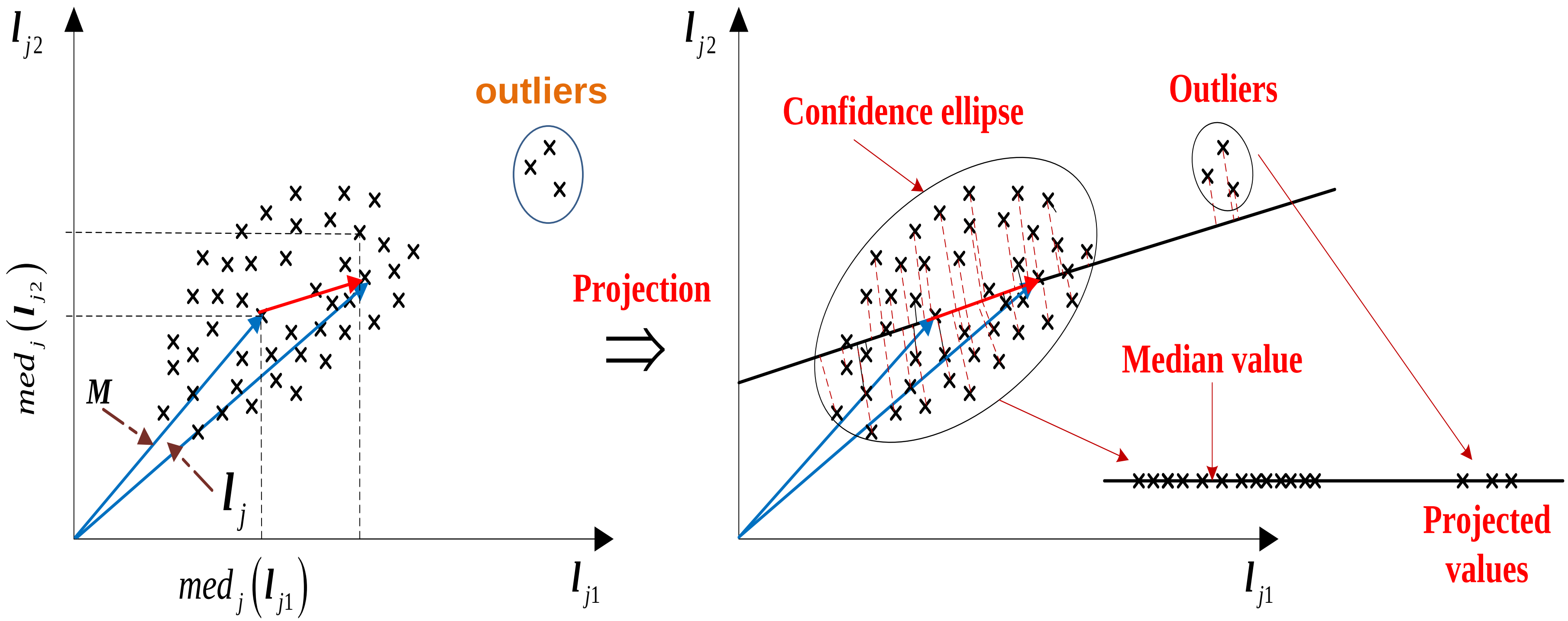
Scatter of matrix Z with outliers

Projection statistic

$$\underline{Z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \\ \mathrm{K} & \mathrm{K} \; \ldots \\ z_{m1} & z_{m2} \end{bmatrix} = \begin{bmatrix} z_1^T \\ z_2^T \\ \mathrm{M} \\ z_m^T \end{bmatrix}$$

$$PS_i = \max_{\|\underline{v}\|=1} \frac{\left| \underline{z}_i^T \underline{v} - \underset{j}{med} \left(\underline{z}_j^T \underline{v} \right) \right|}{1.4826 \; \underset{k}{med} \left| \underline{z}_k^T \underline{v} - med \left(\underline{z}_j^T \underline{v} \right) \right|}$$

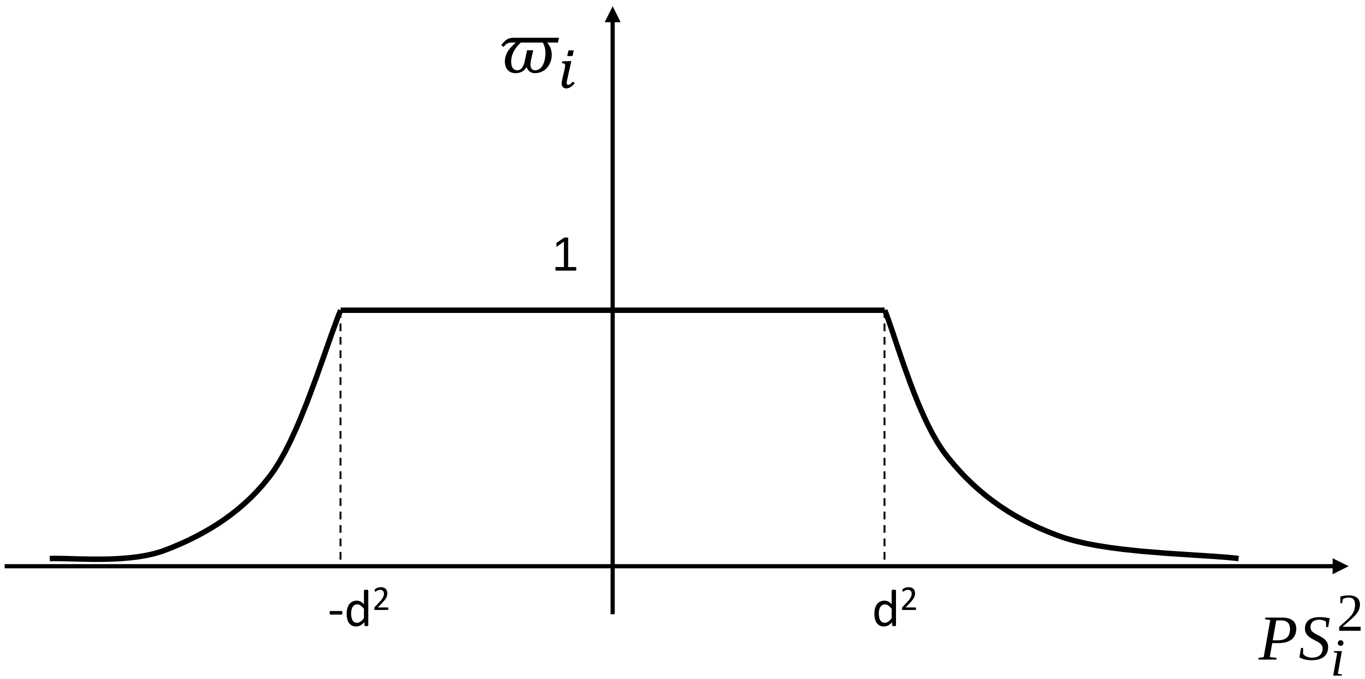
Projection statistic



The Weight Function based on PS

$$\varpi_i = \min\{1, \left(\frac{d}{PS_i}\right)^2\} \quad (8)$$

where $d = \chi^2_{2,0.975}$



Robust Prewhitening

Remember:

$$\mathbb{E}[\tilde{\boldsymbol{e}}_k \tilde{\boldsymbol{e}}_k{}^T] = \begin{bmatrix} \boldsymbol{R}_k + \widetilde{\boldsymbol{R}}_k & 0 \\ 0 & \boldsymbol{\Sigma}_{k|k-1} \end{bmatrix} = \boldsymbol{S}_k \boldsymbol{S}_k{}^T \quad \textbf{(9)}$$

$\boldsymbol{S}_k$ is robust because:

- $\boldsymbol{\Sigma}_{\boldsymbol{k}|\boldsymbol{k}-\boldsymbol{1}}$ **is robust thanks to the robust covariance matrix updating.**

Finally:

$$\underbrace{\boldsymbol{S}_k^{-1} \tilde{\boldsymbol{z}}_k}_{\boldsymbol{y}_k} = \underbrace{\boldsymbol{S}_k^{-1} \widetilde{\boldsymbol{H}}_k}_{\boldsymbol{A}_k} \boldsymbol{x}_k + \underbrace{\boldsymbol{S}_k^{-1} \tilde{\boldsymbol{e}}_k}_{\boldsymbol{\xi}_k} \quad \textbf{(10)}$$

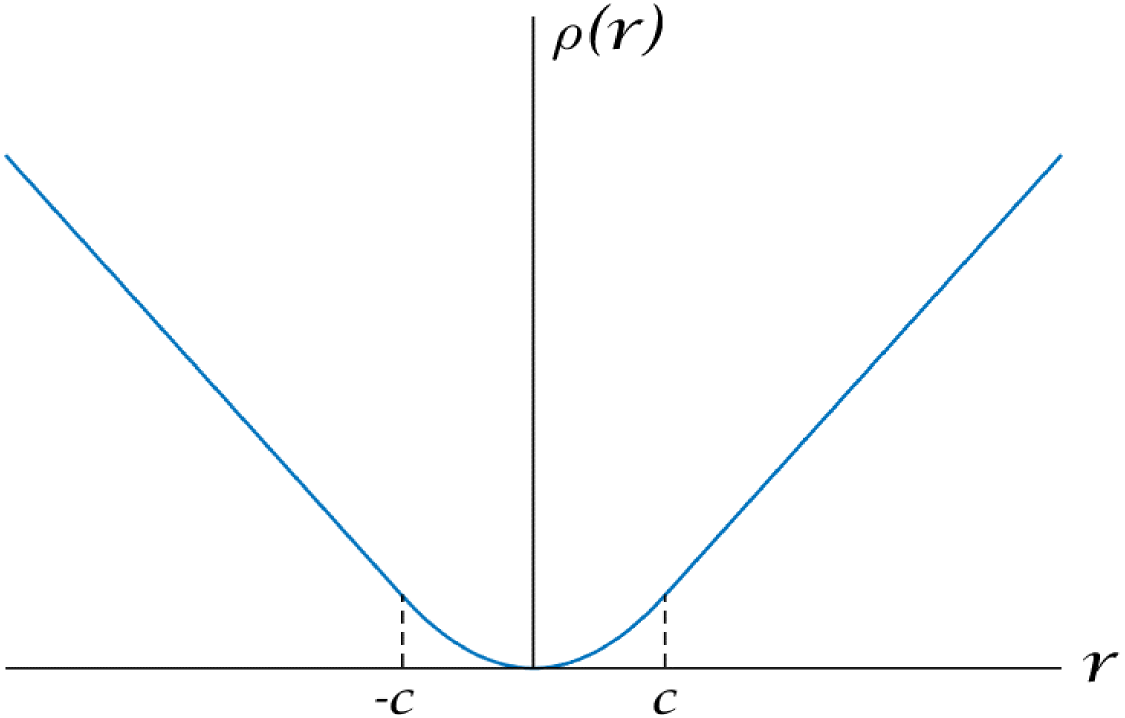
$$\boldsymbol{y}_k = \boldsymbol{A}_k \boldsymbol{x}_k + \boldsymbol{\xi}_k$$

Robust GM-estimator

A GM-estimator minimizes an objective function given by

$$\arg\min J(x) = \sum_{i=1}^{m} \varpi_i^2 \, \rho(r_{Si})$$

Convex Huber ρ-function



Robust Filtering

$$\frac{\partial J(x)}{\partial x} = \sum_{i=1}^{m} -\frac{\varpi_i a_i}{s} \psi(r_{Si}) = 0 \tag{14}$$

- The minimization problem is solved via the **Iteratively Reweighted Least Squares (IRLS)** algorithm:

$$\hat{x}_{k|k}^{(j+1)} = \left(A_k^T Q^{(j)} A_k\right)^{-1} A_k^T Q^{(j)} y_k \tag{15}$$

$$Q = \text{diag}\{q(r_{Si})\} \text{ and } q(r_{Si}) = \psi(r_{Si})/r_{Si}.$$

- Stopping rule:

$$\left\| \hat{x}_{k|k}^{(j+1)} - \hat{x}_{k|k}^{(j)} \right\| < 10^{-2} \tag{16}$$

Robust covariance matrix updating

The estimated state by our GM-UKF tends to a Gaussian distribution asymptotically even when the system process and measurement noise follow a non-Gaussian distribution. Furthermore, the estimation error covariance matrix is updated through

$$\boldsymbol{\Sigma}_{k|k} = 1.0369\left(\boldsymbol{A}_k^T\boldsymbol{A}_k\right)^{-1}\left(\boldsymbol{A}_k^T\boldsymbol{Q}_{\varpi}\boldsymbol{A}_k\right)^{-1}\left(\boldsymbol{A}_k^T\boldsymbol{A}_k\right)^{-1} \quad \textbf{(17)}$$

where $\boldsymbol{Q}_{\boldsymbol{\varpi}} = \mathrm{diag}\{\varpi_{\boldsymbol{i}}^2\}$.

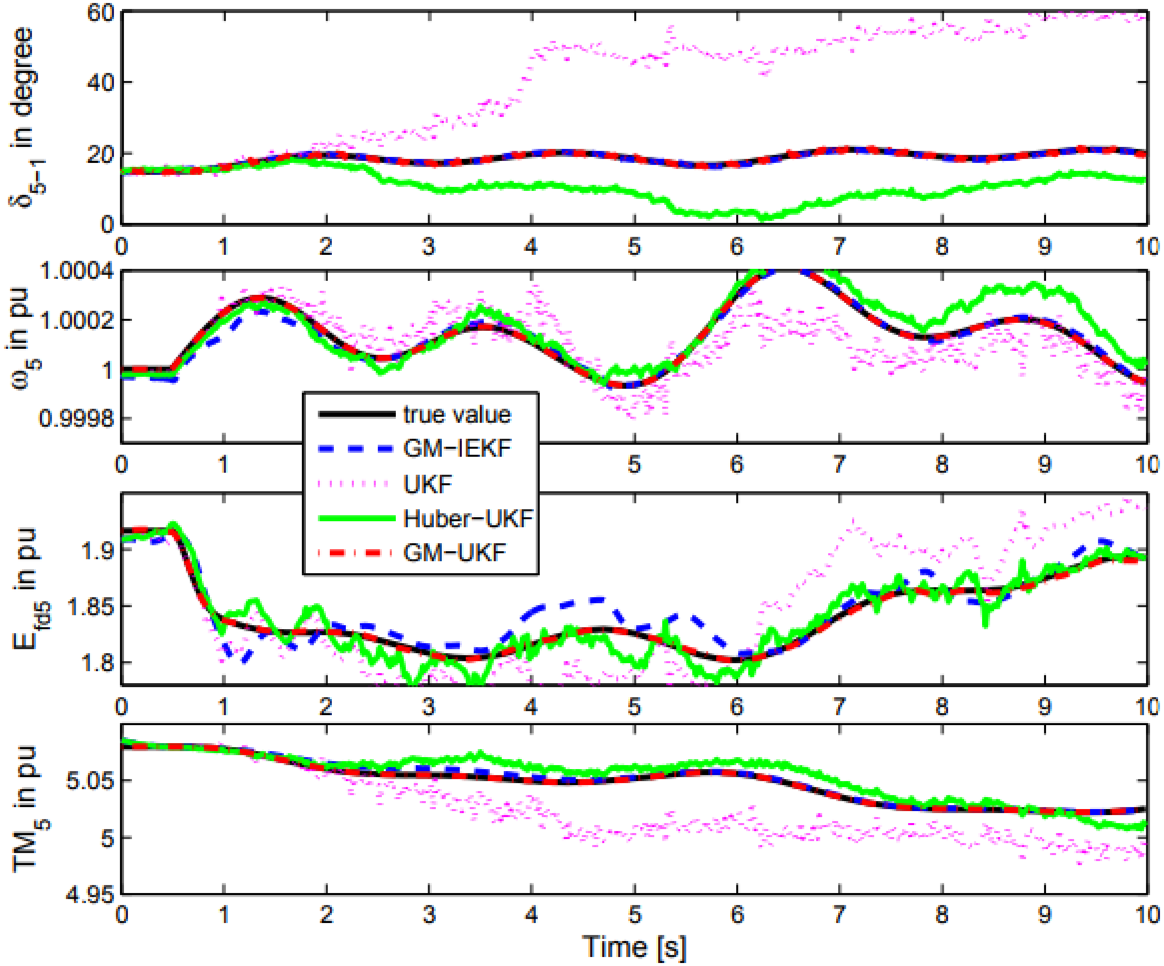
- **$\boldsymbol{Q}_{\varpi}$ determined by PS is used to downweight outliers, yielding robust covariance matrix updating.**

Illustrative Results on IEEE 39-bus system

- **Disturbance:** at t=0.5 seconds, transmission line between buses 15 and 16 is removed.
- **Generator model:** two-axis model with IEEE DC1A excitation system and TGOV1 turbine-governor is assumed.
- **Non-Gaussian noise:**
  - **Bimodal Gaussian mixture** noise with zero mean, variances of 10^{-4} and 10^{-3} and weights of 0.9 and 0.1, respectively, is added to the voltage magnitudes;
  - **Laplacian noise** with zero mean and scale 0.2 is added to the real and reactive power injections.

Case 1: Non-Gaussian Noise

- No outliers;
- Bimodal Gaussian mixture for current and voltage magnitudes;
- Laplace noises for real and reactive power;



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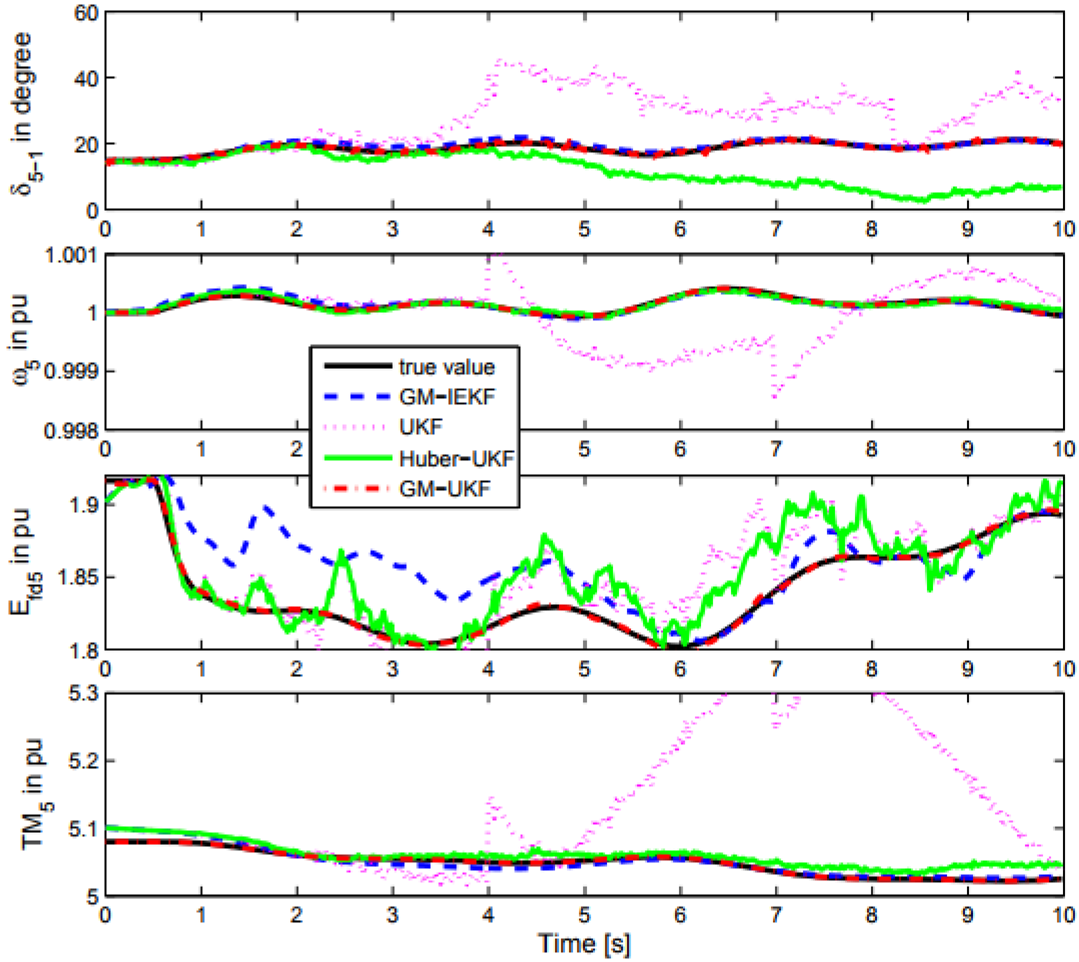
Case 2: Observation Outliers

•The real and reactive power measurements of Generator 5 are corrupted with 20% error from 4s to 6s; Laplace noises for real and reactive power.

- State estimates by UKF are significantly biased;
- GM-UKF achieves much higher statistical efficiency than Huber-UKF and GM-IEKF.

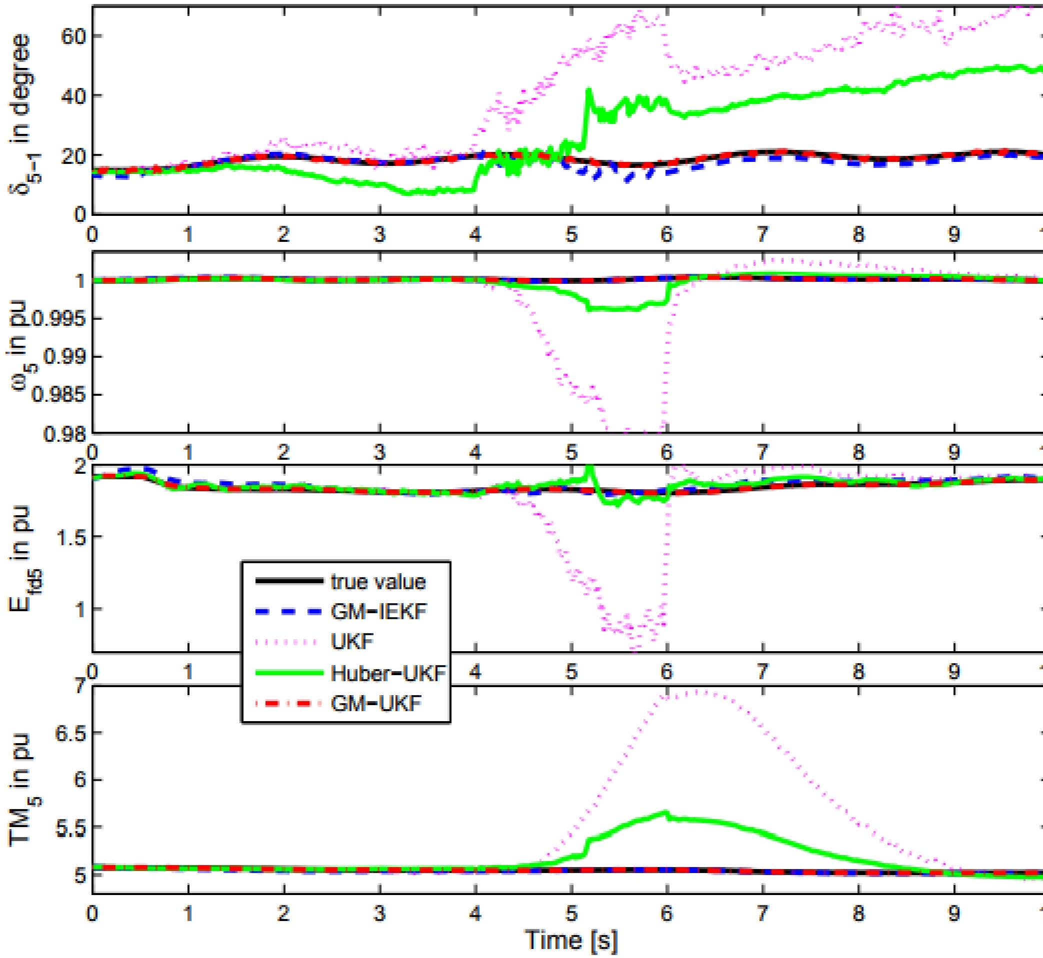


Case 3: Parameter Errors

•The predicted rotor angle of the Generator 5 is incorrect due to the incorrect parameter of G5 from 4s to 6s; Laplace noises for real and reactive power.

•State estimates by UKF and Huber-UKF are significantly;

•GM-UKF achieves much higher statistical efficiency than GM-IEKF.



Breakdown point and computing efficiency

- ✓ Handle **at least 25% outliers** due to cyber attacks, PMU communication issues or model deficiency;
- ✓ Suitable for real-time application.

Table II. Average Computing Time at Each PMU Sample (PC with Intel Core i5, 2.50 GHz, 8GB of RAM)

Cases	EKF	UKF	GM-IEKF	GM-UKF
Case 1	6.24ms	6.28ms	9.64ms	9.52ms
Case 2	6.28ms	6.31ms	9.68ms	9.55ms
Case 3	6.43ms	6.38ms	9.72ms	9.63ms
Case 4	6.45ms	6.40ms	9.71ms	9.62ms
Case 5	6.25ms	6.29ms	9.66ms	9.54ms

Conclusions

- ✓ GM-UKF is able to track power system dynamics even when system nonlinearities are strong;
- ✓ GM-UKF exhibits high statistical efficiency under non-Gaussian noise while being able to suppress observation and innovation outliers;
- ✓ It has high breakdown point (25%) to cyber attacks and model deficiency.
- ✓ Future work will be the development of hybrid DSE that integrates GM-UKF with robust control theory to address model uncertainties.

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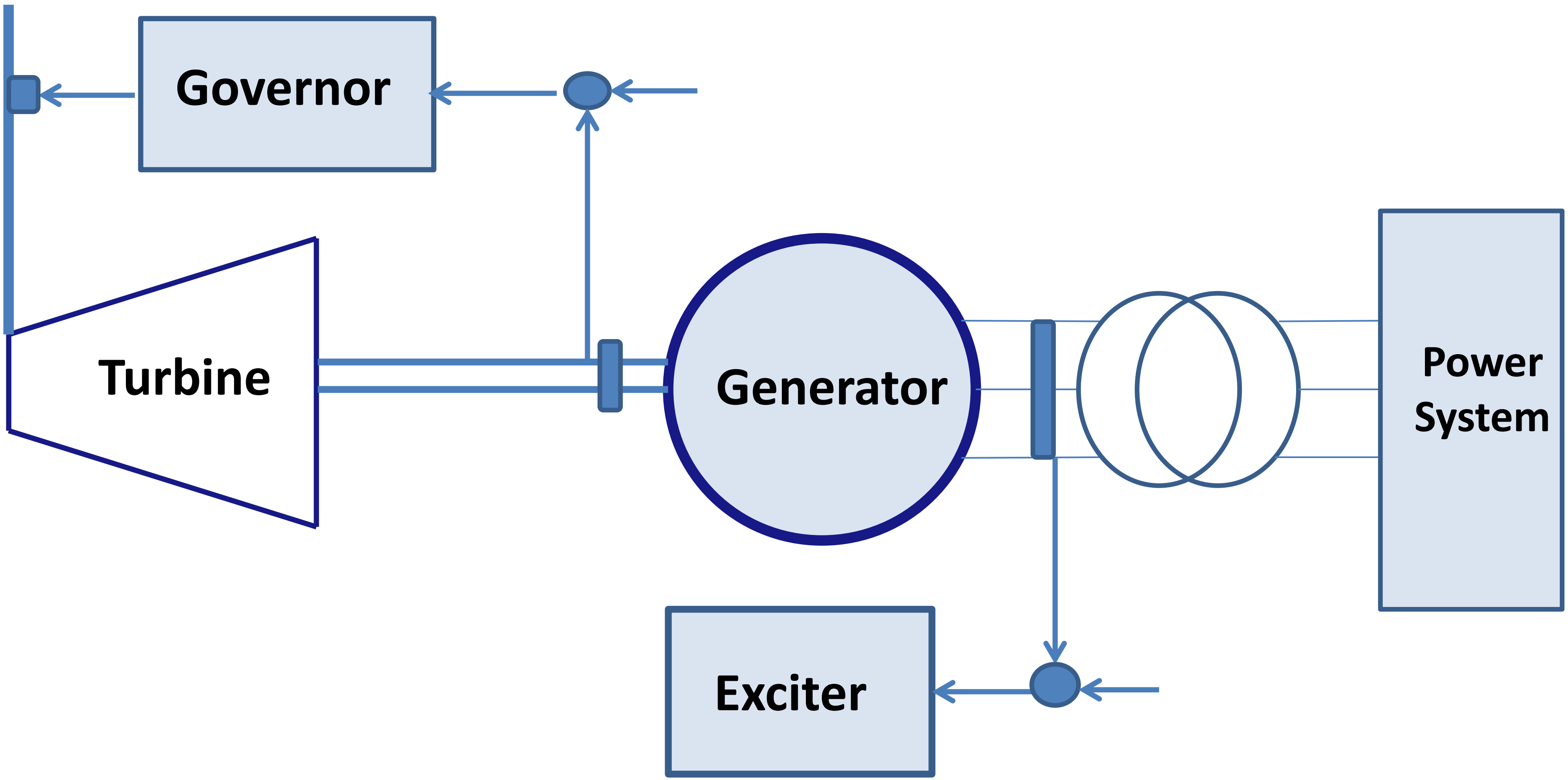
Turbine-Generator System



Two-axis Machine Model

Field winding

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)I_d + E_{fd}$$

Damper winding in the q axis

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)I_q$$

Swing equations

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - E'_d I_d - E'_q I_q - \left(X'_q - X'_d\right)I_d I_q - T_{FW}$$

IEEE DC1A excitation system model

$$T_E \frac{dE_{fd}}{dt} = -\left(K_E + S_E\left(E_{fd}\right)\right) E_{fd} + V_R$$

$$T_F \frac{dR_f}{dt} = -R_f + \frac{K_F}{T_F} E_{fd}$$

$$T_A \frac{dV_R}{dt} = -V_R + K_A R_f - \frac{K_A K_F}{T_F} E_{fd} + K_A\left(V_{ref} - V_t\right)$$

TGOV1 turbine-governor model

$$T_{CH} \frac{dT_M}{dt} = -T_M + P_{SV}$$

$$T_{SV} \frac{dP_{SV}}{dt} = -P_{SV} + P_C - \frac{1}{R_D}\left(\frac{\omega}{\omega_s} - 1\right)$$