Use of PMUs in WLS and LAV Based State Estimation

Ali Abur
Department of Electrical and Computer Engineering
Northeastern University, Boston
abur@ece.neu.edu

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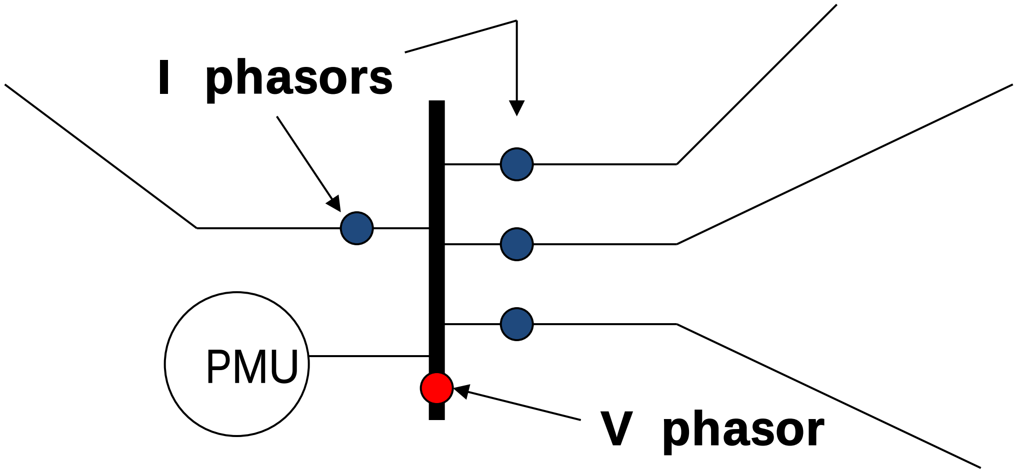
Measurements provided by PMUs



ALL 3-PHASES ARE TYPICALLY MEASURED BUT ONLY POSITIVE SEQUENCE COMPONENTS ARE REPORTED

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = [T] \begin{bmatrix} V_0 \\ V_+ \\ V_- \end{bmatrix} \Rightarrow V_+ = \tfrac{1}{3}[V_A + \propto V_B + \propto^2 V_C]$$

$$\propto = e^{\frac{j2\pi}{3}}$$

Measurement equations

$$SCADA\ Measurements$$

$$Z_S = h(X) + \upsilon \quad Non-linear\ Model$$

$$H_x : \nabla h(X)$$

State *Measurement Error*

$$Phasor\ Measurements$$

$$Z_P = H \cdot X + \upsilon \quad Linear\ Model$$

$$H \ : Function\ of\ network\ parameters\ only$$

A.G. Phadke, J.S. Thorp, and K.J. Karimi, “State Estimation with Phasor Measurements”, IEEE Transactions on Power Systems, vol. 1, no.1, pp. 233-241, February 1986.

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Phasor-only WLS state estimation

$$Z = H \cdot X + \upsilon \quad \textit{Linear Model}$$

WLS state estimation problem:

$$\textit{Minimize} \quad \sum_{i}^{m} \frac{r_i^2}{\sigma_i^2}$$

$$\textit{Subject to} \quad r = Z - H \cdot \hat{X} \quad \textit{residual}$$

$$\hat{X} = G^{-1} H^T R^{-1} Z \quad \textit{Direct solution}$$

$$G = H^T R^{-1} H \; ; R = E\{\upsilon \cdot \upsilon^T\} = \mathrm{cov}(\upsilon)$$

$$\sigma_i^2 : R(i,i) \; \textit{error variance}$$

<u>Non-robust:</u> fails to provide unbiased estimates even when a single bad measurement exists!

Structural simplifications in [G]:

R is assumed to be identity matrix without loss of generality

$$G = H^T \cdot H = \begin{bmatrix} U & \\ & U \\ gA & -bA \\ bA & gA \end{bmatrix}^T \cdot \begin{bmatrix} U & \\ & U \\ gA & -bA \\ bA & gA \end{bmatrix}$$

$$= \begin{bmatrix} U + A^T\left(g^T g + b^T b\right)A & 0 \\ 0 & U + A^T\left(b^T b + g^T g\right)A \end{bmatrix}$$

[G] matrix:

- Is block – diagonal
- Has identical diagonal blocks
- Is constant, independent of the state

Correction for shunt terms:

$$[Z] = (H + H_{sh}) \cdot X + \upsilon = H \cdot X + u$$

$$u = H_{sh} \cdot X + \upsilon$$

$$E\{u\} = H_{sh} \cdot E\{X\}$$

$$E\{X\} = \hat{X} = G^{-1} H^T R^{-1} Z$$

$$X^{corr} = G^{-1} H^T R^{-1} (Z - H_{sh} \cdot \hat{X})$$

$$= \hat{X} - \underbrace{G^{-1} H^T R^{-1} H_{sh}}_{\text{Very sparse}} \cdot \hat{X}$$

Fast Decoupled WLS Implementation Results

MEAN CPU TIMES OF 100 SIMULATIONS

System	Case	CPU Times (ms)	
		WLS	Decoupled WLS
159-bus	1	5	2.4
	2	5.7	2.7
	3	9.3	3.9
265-bus	1	7.5	3.5
	2	8.7	3.9
	3	14.8	5.8
3625-bus	1	137.4	75.9
	2	169.5	95.7
	3	284.7	165.6

CASE 1: NO BAD DATA, CASE 2: 1 BAD DATA, CASE 3: 5 BAD DATA

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L_1 /Least Absolute Value (LAV) Estimator

Minimizing the L_1-norm of residuals

$$Minimize \sum_{i=1}^{m} |r_i|$$

$$Subject\ to\ \ Z = H \cdot \hat{X} + r$$

Robust up to a limited number of existing bad data, but vulnerable to **leverage** measurements[*].

Scaling can be used to eliminate leverage measurements.

[*] Mili L., Cheniae M.G., and Rousseeuw P.J., "Robust State Estimation of Electric Power Systems" IEEE Transactions on Circuits and Systems, Vol. 41, No. 5, May 1994, pp.349-358.

Definitions:

- What are leverage measurements?

Certain types of measurements due to their type and location will move the estimates towards wrong values when they carry bad data. They are called “leverage” measurements.

- What is scaling?

Multiplying measurement equations by appropriate constants to eliminate “leverage measurements”.

Why was LAV not used before with SCADA measurements?

- SCADA measurements

→Nonlinear measurement equations

→Iterative solution (higher cpu burden)

Why LAV is preferable now?

- Major vulnerability of LAV is the so called "leverage measurements". When PMU measurements are used it is possible to eliminate them by simple scaling.

- Recent advances in efficient Linear Programming (LP) code enable implementation for large scale systems.

How does L_1 estimator tell which measurement is good which is bad?

- L1 estimator has the "interpolation" property, i.e. it fits the solution to a minimum number of required measurements for which the corresponding residual vector will have the minimum L1 norm.
- As a result, bad observation points (bad measurements) will automatically be left out during the solution.

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Bad data processing

WLS: Post estimation bad data processing / re-estimation

$\Omega = R - HG^{-1}H^T$ Normalized Residuals Test (NRT):

$G = H^T R^{-1} H$

$r_i^N = |r_i| / \sqrt{\Omega_{ii}}$ → $z_i^{new} = z_i^{bad} - R_{ii} / \Omega_{ii} \, r_i^{bad}$ → Update the estimates

NRT is repeated as many times as the number of bad data.

L_1: Linear Programming Solution

LP problem is solved once. Choice of initial basis impacts solution time/iterations.

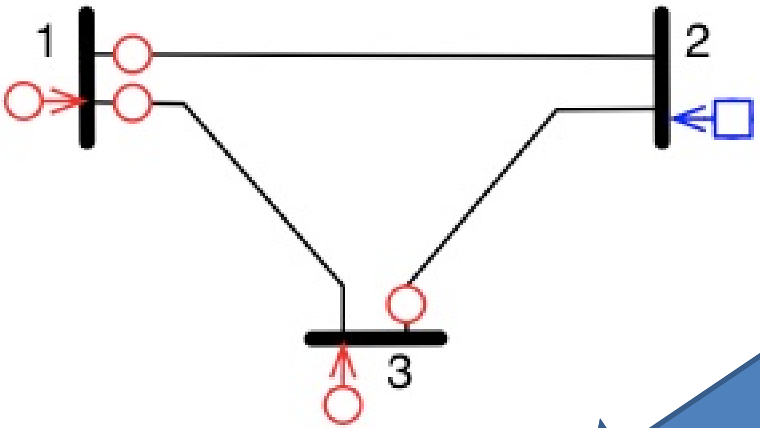
Simulation Results

3625 bus, 4836 branch utility system

	CPU Time (seconds)		
	No Bad Data	Single Bad Data	Five Bad Data
LAV with built-in BD removal	3.33	3.36	3.57
WLS using post-SE BD detection test	2.32	9.38	50.2

- Bad data handling of LAV solver remains fairly insensitive to the number of bad data.
- Bad data handling of WLS solver will be proportionally slower with increasing number of bad data in the measurement set.

SCADA Based Implementation



Measurement set:

- Power injection measurements
- Power flow measurements
- Voltage magnitude measurements

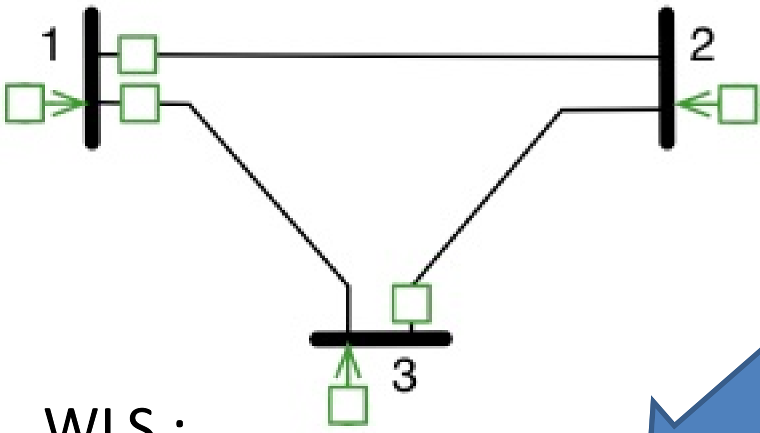
Weighted Least Squares (WLS) Estimator

- Well-developed and widely-known
- Requires bad-data analysis
  - Normalized residuals test
  - Re-weighting

Least Absolute Value (LAV) Estimator

- Linear programming based, requires more cpu time for solution.
- Does not require bad-data analysis
- Deficiency in the presence of leverage measurements

PMU Based Implementation



Phasor Measurement Units (PMUs):

Linearly related states and measurements.

WLS :

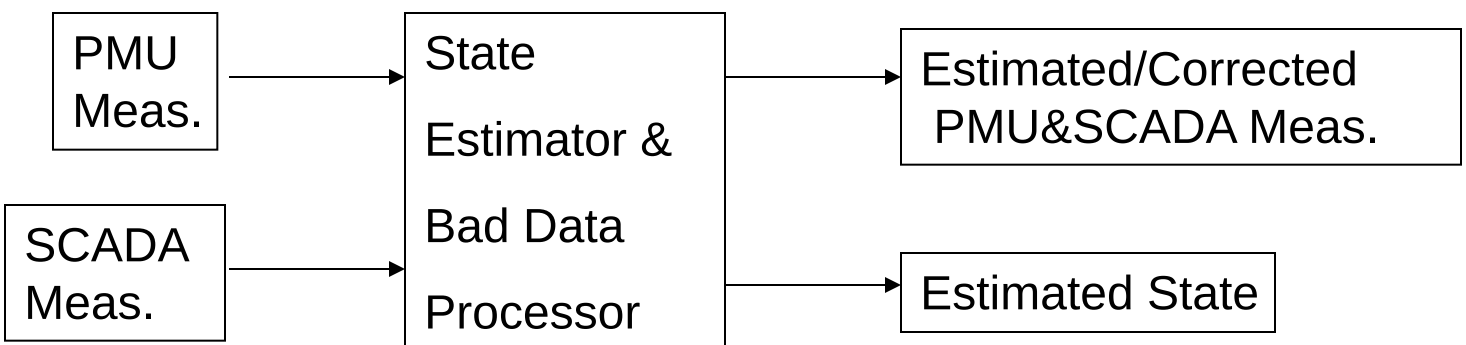
- Linear solution
- *Requires bad-data analysis*
  - Normalized residuals test
  - Re-weighting (not applicable)
- No deficiency in the presence of leverage measurements, with *scaling*.
- Exact cancellations in [G]

LAV (robust):

- Linear programming (single step), *computationally competitive* with WLS.
- *Does not* require bad-data analysis
- No deficiency in the presence of leverage measurements, with *scaling*.

Linear Estimator Implementation: No need for a reference bus

- Eliminate the reference phase angle from the SE formulation.
- Bad data in SCADA as well as phasor measurements can be detected and identified with sufficiently redundant measurement sets.



Final Remarks

- Some of the previously disregarded robust estimators may be worth re-considering when formulating phasor-only estimation.
- LAV estimator is more robust compared to WLS estimator when PMU measurements are used exclusively.
- Computational performances of the two estimators are comparable but favors LAV in the presence of many bad data.

Thank you !