# The Impact of Pseudo-Measurements on State Estimator Accuracy

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# Power System State Estimation Overview

- The static state of an power system network is

$$x = \begin{pmatrix} \theta \\ \phi \\ v \\ \delta \end{pmatrix} \in \mathbb{R}^n$$

where $\theta$ is a vector of bus voltage angles, $\phi$ is a vector of transformer phase shifts, $v$ is a vector of bus voltage magnitudes and $\delta$ is a vector of transformer off-nominal voltage ratios.

# Power System State Estimation Overview

- Network parameters and topology are assumed to be perfectly known in the state estimation problem formulation.
- The available real-time measurements typically include:
  - real and reactive line flows
  - real and reactive bus injections
  - bus voltage magnitudes
  - line flow current magnitudes
  - PMU phasor measurements

# Power System State Estimation Overview

- When the set of available real-time measurements is sufficient to allow a calculation of the system state vector, the system is said to be *observable.*
- Otherwise the system is *unobservable*.
- When the system is unobservable, there are observable islands as well as unobservable regions within the network.

# Power System State Estimation Overview

- The unobservable regions of the network can be estimated by using pseudo-measurements to augment the available real-time measurements.
- Pseudo-measurements are typically calculated using short-term load forecasts or historical data.
- Pseudo-measurements are much less accurate than the real-time measurements.

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# Power System State Estimation Overview

- A state estimate is typically used to estimate line flows throughout the network.
- These data, in turn, are used to assess system security.
- The state estimate is used for real-time contingency analysis.
- In this paper, which is tutorial in nature, we discuss the effect of pseudo-measurement errors on the accuracy of state, line flow and bus injection estimates.

- The measurements, $z \in \mathbb{R}^m$, are modeled by the nonlinear measurement equation

$$z = h(x) + \epsilon$$

  where $\epsilon$ is a vector of measurement errors.
- Measurement error $\epsilon_i$ is modeled as a zero mean random variable whose standard deviation is $\sigma_i$.

- The power system state estimation problem is usually formulated as a nonlinear weighted least-squares (WLS) problem in which we seek to compute $x$ to minimize the objective function:

$$f\left(x\right) = \frac{1}{2}\left[z - h\left(x\right)\right]^{T} R^{-1}\left[z - h\left(x\right)\right]$$

where $R$ is the $m \times m$ diagonal matrix

$$R = \begin{pmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_m^2 \end{pmatrix}$$

- Newton's method is widely used to solve systems of nonlinear equations.
- When applied to the nonlinear least-squares problem it leads to the following iterative solution:

$$x^{k+1} = x^k + B_k^{-1} H_k r_k$$

where

$$\begin{aligned} r_k &= z - h\left(x^k\right) \\ H_k &= H\left(x^k\right) = \nabla h\left(x^k\right) \end{aligned}$$

and the Hessian matrix $B_k$ is given by

$$B_k = H_k^T R^{-1} H_k + \sum_{i=1}^{m} r_i^2 R^{-1} \nabla^2 r_i$$

- Often, the second term in the above expression for $B_k$ is ignored leading to the Gauss-Newton method.

- In power system state estimation, the nonlinear least-squares problem is typically solved using the Gauss-Newton method which leads to the following iterative solution:

$$x^{k+1} = x^k + G_k^{-1} H_k r_k$$

where

$$G_k = H_k^T R^{-1} H_k$$

- The Gauss-Newton method ignores the second order information contained in $r_i^2 R^{-1} \nabla^2 r_i$.
- The term $r_i^2 R^{-1} \nabla^2 r_i$ will be small provided $r_i^2 / \sigma_i^2$ is small.

# Network Observability and Critical Measurements

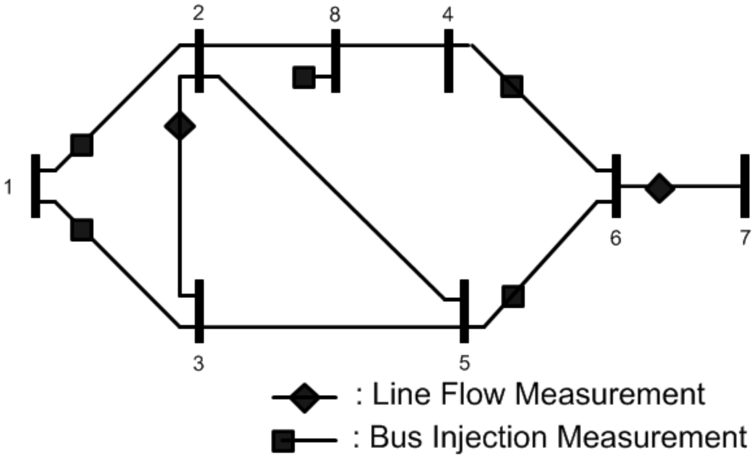
- When a network is observable, $G_k$ is invertible which, in turn requires that the measurement Jacobian matrix, $H_k$ have full rank.
- Network observability is determined by the type and location of the measurements in the network as well as the network topology.
- Observability analysis can be done by direct evaluation of the rank of $G_k$, or by topological analysis of the measured network.
- A *critical measurement* is one whose loss decreases the rank of $G_k$.

# Network Observability and Critical Measurements

- The result of observability analysis is to identify islands of observability as well as unobservable branches.
- A by-product of observability analysis is the identification of critical measurements.
- As an example, consider the measured network shown on the following slide.

# Network Observability and Critical Measurements

## Example: An Eight Bus Measured Network



- Six line flow measurements and one injection measurement are shown on the previous slide.
- There are also two bus voltage magnitude measurements, one at bus 1 and one at bus 6, that are not shown on the one line diagram.
- Observability analysis of this network reveals that there are two observable islands and four unobservable branches.
- The observable islands and unobservable branches are shown on the following slide.

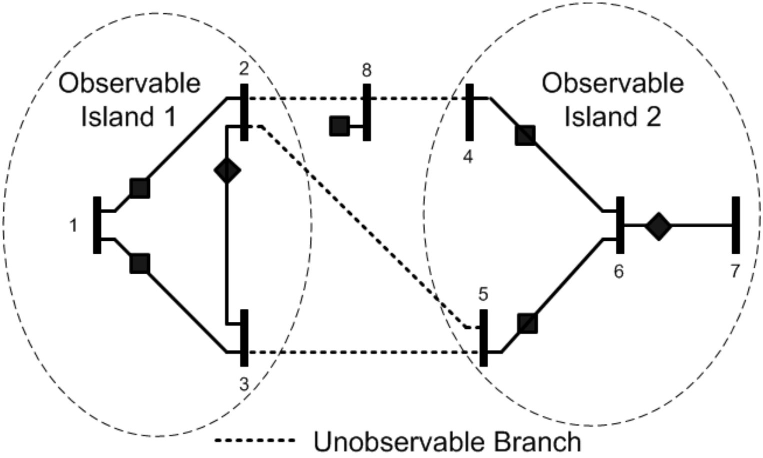



Figure: Observable Islands for the Eight Bus Example

- In this example, at least one additional measurement, a pseudo-measurement, involving an unobservable branch power flow is required in order to attain observability.
- This pseudo-measurement could be a bus injection measurement at either bus 2, or 3, or 4, or 5.

- In this example, the measurements are partitioned into three sets;

$$z = \begin{pmatrix} z_1 \\ z_2 \\ z_C \end{pmatrix} = \begin{pmatrix} h_1(x_1) + \epsilon_1 \\ h_2(x_2) + \epsilon_2 \\ h_C(x_1, x_2, x_3) + \epsilon_C \end{pmatrix}$$

- The critical measurement $z_C$ is the bus injection at bus 8 which is incident to unobservable branches.

- In the general case, when a network is unobservable, there will be k observability islands and the measurement structure is given by

$$z = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_k \\ z_C \end{pmatrix} = \begin{pmatrix} h_1(x_1) + \epsilon_1 \\ h_2(x_2) + \epsilon_2 \\ \vdots \\ h_k(x_k) + \epsilon_k \\ h_C(x_1, x_2, \dots, x_k) + \epsilon_C \end{pmatrix}$$

- The measurement set $z_C$ consists of measurements that are incident to unobservable branches.

# The Branch Angle Transformation

- When multiple observable islands occur, it is necessary to choose a reference bus in each island when the state vector consists of bus voltage angles and bus voltage magnitudes.
- In order to avoid this complication, we can instead choose a set of $N-1$ branch angles to replace the bus voltage angles as state variables.
- The branch angles are related to the bus voltage angles by

$$\theta_B = A^T \theta$$

where $A$ is the network node-to-branch incidence matrix.

# The Branch Angle Transformation

- Let $T$ be a set of $N-1$ branches that form a spanning tree of the network, then we can partition $\theta_B$ into

$$\theta_B = \begin{pmatrix} \theta_T \\ \theta_L \end{pmatrix}$$

- $\theta_T$ is sufficient to define all branch angles since the sum of branch angles around a fundamental loop must be 0. Therefore

$$\theta_L = -B_T \theta_T$$

# The Branch Angle Transformation

- Replacing $\theta$ by $\theta_T$ in the state vector results in a bordered block diagonal structure for the measurement Jacobian matrix:

$$H = \begin{pmatrix} H_{11} & 0 & \cdots & 0 & 0 \\ 0 & H_{22} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & H_{kk} & 0 \\ H_{C1} & H_{C2} & \cdots & H_{Ck} & H_{CC} \end{pmatrix}$$

where

$$H_{ii} \in \mathbb{R}^{m_i \times n_i} \quad \text{and} \quad H_{CC} \in \mathbb{R}^{m_c \times n_c}$$

- The diagonal blocks $H_{11}, \ldots, H_{kk}$ have full column rank provided that each observable island contains at least one voltage magnitude measurement.

# Critical Pseudo-Measurements

- The rank of $H_{CC}$ is $m_c$, the number of critical measurements incident to unobservable branches.
- In order to attain observability, it is necessary to add $n_c - m_c$ suitable pseudo-measurements.
- When the measurement set is augmented by critical pseudo-measurements in this manner, $H_{CC} \in \mathbb{R}^{n_c \times n_c}$ is an invertible matrix.

# Error Analysis

- The WLS objective function can be written as

$$f(x) = \frac{1}{2}\sum_{i=1}^{k} r_i(x_i)^T R_{ii}^{-1} r_i(x_i) + \frac{1}{2} r_C(x)^T R_C^{-1} r_C(x)$$

- It is well known that the measurement residuals for critical measurements are zero. Consequently, the WLS objective function reduces to

$$f(x) = \frac{1}{2}\sum_{i=1}^{k} r_i(x_i)^T R_{ii}^{-1} r_i(x_i)$$

- The state estimates for the observable islands satisfy the first- order optimality conditions:

$$H_{ii} R_{ii}^{-1} \left[z_i - h_i(x_i)\right] = 0$$

- The state estimate for the unobservable part of the network satisfies

$$h_C(x_C) = z_C$$

# Error Analysis

- Using the linearizing approximations

$$
\begin{aligned}
h_i\left(\widehat{x}_i\right) &\approx h_i\left(x_i\right) + H_{ii}\left(\widehat{x}_i\right)\left(\widehat{x}_i - x_i\right) \\
h_C\left(\widehat{x}\right) &\approx h_C\left(x\right) + H_C\left(\widehat{x}\right)\left(\widehat{x} - x\right)
\end{aligned}
$$

we see that

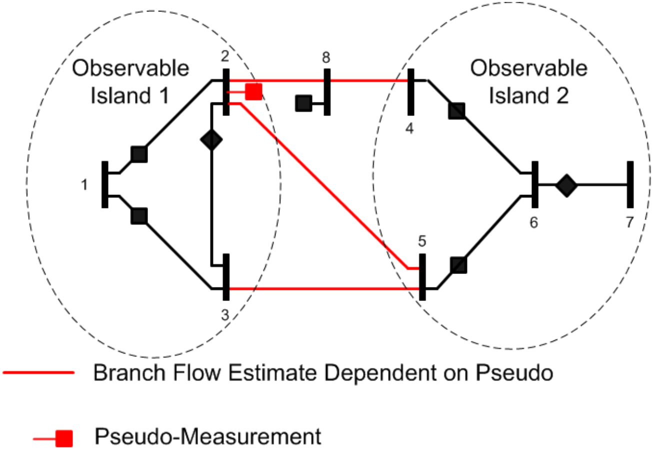
$$
\widetilde{x}_i = -G_{ii}\left(\widehat{x}_i\right)^{-1} H_{ii}^T\left(\widehat{x}_i\right) R_i^{-1}\epsilon_i \quad i = 1, \ldots, k
$$

and

$$
\widetilde{x}_C = -H_{CC}^{-1}\left(\epsilon_C + \sum_{i=1}^{k} H_{Ci}\widetilde{x}_i\right)
$$

where $\widetilde{x}_i = x_i - \widehat{x}_i$ and $\widetilde{x}_C = x_C - \widehat{x}_C$.

- Shown below is the eight bus network with one pseudo-measurement added at bus 2.

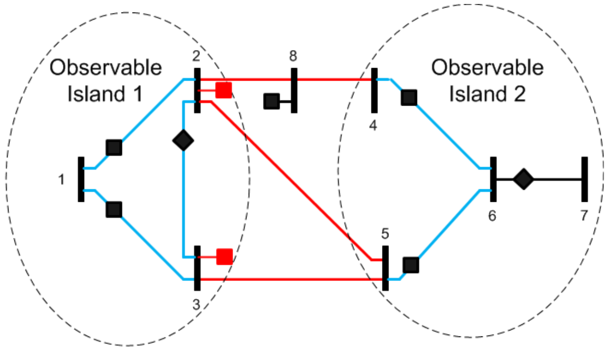


- In this example, neither observable island state estimate is affected by the pseudo-measurement.
- The unobservable branches, of course, must use the pseudo-measurement data to calculate line flows and thus their accuracy is low compared with that of the observable islands.

# Error Analysis

## Example

- Shown below is the eight bus network with pseudo-measurements added at buses 2 and 3.




Branch Flow Estimate Dependent on Pseudos

Branch Flow Estimate Influenced by Pseudos

Pseudo-Measurement

# Error Analysis

- **As seen from the above error analysis, if only critical pseudo-measurements are used, the pseudo-measurement errors do not affect the accuracy of the state estimates for the observable islands.**
- **If additional pseudo-measurements are used then pseudo-measurement errors will affect the accuracy of the state estimates in some or all of the observable islands.**
- **Furthermore, if only critical pseudo-measurements are used, all pseudo-measurement residuals will be zero.**
- **Large pseudo-measurement residuals may cause the Gauss-Newton approximation to the Hessian matrix to be poor and thereby contribute to slow convergence or non-convergence.**
- **This effect can be ameliorated by assigning large $\sigma_i$'s to the pseudo-measurements.**