Network Observability Analysis

Fully observable network:

A power system is said to be fully observable if voltage phasors at all system buses can be uniquely estimated using the available measurements.

Unobservable branch:

If the system is found not to be observable, it will imply that there are unobservable branches whose power flows cannot be determined.

Observable island:

Unobservable branches connect observable islands of an unobservable system.

State of each observable island can be estimated using any one of the buses in that island as the reference bus.
Network Observability Analysis

Given the set of measurements and the network topology, the system will be observable if the system state can be uniquely calculated.

Consider the linearized measurement model:

$$\Delta z = H \Delta x + e$$

The WLS estimate $\Delta \hat{x}$ will be given by:

$$\Delta \hat{x} = \left( H^T R^{-1} H \right)^{-1} H^T R^{-1} \Delta z$$

A unique solution for $\Delta x$ can be calculated if $(H^T R^{-1} H)$ is nonsingular or equivalently if $H$ has full column rank, i.e. rank$[H] = n$, where $n$ is the total number of states.

Using the weak coupling between $P - V$ and $Q - \theta$, the linearized model can be decoupled as follows:

$$\Delta z_A = H_{AA} \Delta \theta + e_A$$
$$\Delta z_R = H_{RR} \Delta V + e_R$$

and the $P - \theta$ and $Q - V$ observability can be separately tested.
Steps of Observability Analysis

Observability analysis involves the following steps:

1. Determine if the system is fully observable. If found observable, state estimation solution will be initiated using the existing measurements and information on network topology.

2. If found unobservable, identify all unobservable branches and the observable islands that they connect.

3. Place pseudo-measurements at appropriate locations to merge all observable islands into a single fully observable system. These pseudo-measurements will be based on the load forecasts and scheduled generation.

Network observability analysis can be carried out based on either numerical or topological methods.
DC Power Flow Model

Note that system observability is independent of:

- the branch parameters, and
- the operating state of the system.

Thus, without loss of generality, one can assume that all system branches have an impedance of $j1.0$ p.u. and all bus voltage magnitudes are equal to 1.0 p.u.

Power flow through a branch $km$:

$$P_{km} = \frac{V_k V_m}{X_{km}} \sin \theta_{km}$$

Taylor expansion around $V_k^0 = V_m^0 = 1.0$ and $\theta_k^0 = 0$:

$$P_{km} \approx P_{km}^0 + \frac{V_k^0 V_m^0}{X_{km}} \cos \theta_{km}^0 (\theta_{km} - \theta_{km}^0)$$

or

$$P_{km} = \theta_{km} = \theta_k - \theta_m$$
D.C. Branch Flows

Then the d.c. power flows along all the system branches can be written as:

\[ P_b = C\theta \]

where: \( P_b \) is the vector of branch flows  
\( C \) is the reduced branch-bus incidence matrix  
\( \theta \) is the vector of bus voltage phase angles

Arbitrarily assigning directions to all system branches, the full branch to bus incidence matrix \( C_F \) can be formed as follows:

\[ C_F(i, j) = \begin{cases} 
1 & \text{if bus } j \text{ is the sending end of branch } i \\
-1 & \text{if bus } j \text{ is the receiving end of branch } i \\
0 & \text{otherwise} 
\end{cases} \]

The matrix formed by deleting any one of the columns of \( C_F \) is called the reduced branch-bus incidence matrix and simply denoted by \( C' \).

The column corresponding to the reference bus is the one which is commonly chosen for deletion.
Example 2:

Consider the network and its directed graph shown below.

Full branch to bus incidence matrix $C_F$:

\[
C_F = \begin{bmatrix}
1 & -1 & 1 & -1 & 1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 
\end{bmatrix}
\]
Logic Behind Numerical Observability Analysis

As derived above, the branch d.c. power flows can be written in compact form as follows:

\[ P_b = C\theta \]

Consider the special case where all system measurements (flows and injections) are equal to zero. Then, the following statements will be true:

- If the system is fully observable, there will be no unobservable branches. Hence, every branch flow must be equal to zero.

- If a system state can be found where one or more branch flows will be non-zero, then this state will be an unobservable state. Those branches with non-zero flows will be unobservable branches.

If all measurements are zero, none of the branch flows can be nonzero for a fully observable system. If any of the branch flows are nonzero, the corresponding branches will be unobservable.
Numerical Approach to Observability Analysis

Using the decoupled linear measurement model:

\[ H_{AA} \theta = z_A \]

the WLS estimate for \( \theta \) will be given by:

\[ \hat{\theta} = \left( H_{AA}^T H_{AA} \right)^{-1} H_{AA}^T z_A \]

If \( H_{AA} \hat{\theta} = 0 \) and \( P_b = C \hat{\theta} \neq 0 \),

then \( \hat{\theta} \) will be called an unobservable state.

If \( P_b(i) \neq 0 \) for a branch \( i \),

then it will be called an unobservable branch.

Branches having no incident measurements, are called the irrelevant branches. The estimated state will be independent of the status (on/off) and parameters of these branches and therefore they can be disregarded for network observability analysis purposes.
Determining the Unobservable Branches

Let us again consider the decoupled linearized model:

\[(H^T_{AA} H_{AA})\hat{\theta} = 0\]
\[G_{AA} \hat{\theta} = 0\]

If \(G_{AA}\) is nonsingular, the system will be observable.

If \(G_{AA}\) is singular, then row/column permutations can be used to reorder and partition the matrix as follows:

\[
\begin{bmatrix}
G_{11} & G_{12} \\
G_{21} & G_{22}
\end{bmatrix}
\begin{bmatrix}
\hat{\theta}_a \\
\hat{\theta}_b
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

where \(G_{11}\) is a nonsingular submatrix within \(G_{AA}\).

By assigning arbitrary but distinct values to \(\hat{\theta}_b\) entries as \(\overline{\theta}_b\), one of many possible solutions for \(\hat{\theta}_a\) can be obtained as:

\[
\hat{\theta}_a = -G_{11}^{-1} G_{12} \overline{\theta}_b
\]

The branch flows corresponding to this solution \((\hat{\theta}_a, \overline{\theta}_b) = \hat{\theta}^*\) can then be found as:

\[C \hat{\theta}^* = P_b^*\]

Those branches \(i\) with \(P_b^*(i) \neq 0\) will be identified as the unobservable branches.
Determining the Unobservable State $\hat{\theta}^*$

In practice, the unobservable state is determined by partial factorization of $G_{AA}$ and simultaneous modification of the right hand side zero vector:

$$G_{AA} \hat{\theta} = 0$$

If a zero pivot is encountered during the factorization of $G_{AA}$, it will be replaced by 1.0. Furthermore, an arbitrary integer will be assigned to the corresponding row entry in the right hand side vector.

In order to ensure that no two entries are assigned the same integer value, they are assigned in increasing order by increments of 1 starting from 0.

This way, even for singular $G_{AA}$, factorization can be completed. Note that:

- The arbitrarily assigned integer values to the right hand side vector correspond to $\overline{\theta}_b$ in the compact description given above.
- The solution of the system subsequent to the above described factorization of $G_{AA}$ will yield the unobservable state $\hat{\theta}^*$ as described in compact form above.
Numerical Observability Analysis Algorithm:

1. Remove all irrelevant branches.

2. Form the decoupled linearized gain matrix for the $P-\theta$ estimation problem:

$$G_{AA} = H_{AA}^T R_A^{-1} H_{AA}$$

3. If $G_{AA}$ is nonsingular, the system will be declared fully observable.
   Else, the unobservable branches will be found as described above.

4. Remove the unobservable branches and all injections that are incident at the unobservable branches.

5. Go to step 1.
Example 3:

Note that branch 1-3 is an irrelevant branch.

\[
G_{AA} = \begin{bmatrix}
2 & 0 & 0 & -3 & 0 & 1 \\
0 & 2 & 0 & -3 & 0 & 1 \\
0 & 0 & 1 & 0 & -1 & 0 \\
-3 & -3 & 0 & 9 & 0 & -3 \\
0 & 0 & -1 & 0 & 2 & -1 \\
1 & 1 & 0 & -3 & -1 & 2 \\
\end{bmatrix},
\]

\[
H_{AA} = \begin{bmatrix}
-1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 \\
-1 & -1 & 0 & 3 & 0 & -1 \\
\end{bmatrix}
\]
Reordering the rows/columns of $G_{AA}$ so that

$$\theta_a^T = [\theta_1 \ \theta_2 \ \theta_3 \ \theta_5]$$

$$\theta_b^T = [\theta_6 \ \theta_4]$$

The reordered matrix $G_{AA}^{ord}$ will then be:

$$G_{AA}^{ord} = \begin{bmatrix}
2 & 0 & 0 & 0 & 1 & -3 \\
0 & 2 & 0 & 0 & 1 & -3 \\
0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & -1 & 2 & -1 & 0 \\
1 & 1 & 0 & -1 & 2 & -3 \\
-3 & -3 & 0 & 0 & -3 & 9 \\
\end{bmatrix}$$

Let $\theta_6 = 1$, $\theta_4 = 0$, then solve for the rest as:

$$\theta_1 = -0.5,$$
$$\theta_2 = -0.5,$$
$$\theta_3 = 1.0,$$
$$\theta_5 = 1$$
Calculating the resulting branch flows:

<table>
<thead>
<tr>
<th>Branch</th>
<th>Calculated Flow</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>0</td>
<td>observable</td>
</tr>
<tr>
<td>1-4</td>
<td>-0.5</td>
<td>unobservable</td>
</tr>
<tr>
<td>2-4</td>
<td>-0.5</td>
<td>unobservable</td>
</tr>
<tr>
<td>4-6</td>
<td>-0.5</td>
<td>unobservable</td>
</tr>
<tr>
<td>3-5</td>
<td>0</td>
<td>observable</td>
</tr>
<tr>
<td>5-6</td>
<td>0</td>
<td>observable</td>
</tr>
<tr>
<td>1-3</td>
<td>-0.5</td>
<td>unobservable</td>
</tr>
</tbody>
</table>

Remove all unobservable branches and the injection at bus 4.

Therefore, three observable islands will be identified as in the above figure, i.e. branch 1-2, branches 3-5, 5-6, and the isolated bus 4.
Measurement Placement

Observability can be restored by merging islands. Candidate measurements that can merge islands are:

- the line flows along branches that connect observable islands, and
- the injections at the boundary buses of observable islands.

Example 4:

Previous observability analysis identified 5 observable islands defined by buses [3 4], [1], [2], [5], and [6].
**Step 1** Form the Jacobian matrix $H$ and the gain matrix $G$:

$$H = \begin{bmatrix}
-1 & -1 & 3 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & -1 & 3 & -1 & -1
\end{bmatrix}$$

$$G = \begin{bmatrix}
1 & 1 & -3 & 1 & 0 & 0 \\
1 & 1 & -3 & 1 & 0 & 0 \\
-3 & -3 & 11 & -7 & 1 & 1 \\
1 & 1 & -7 & 11 & -3 & -3 \\
0 & 0 & 1 & -3 & 1 & 1 \\
0 & 0 & 1 & -3 & 1 & 1
\end{bmatrix}$$

Triangular factors of $G$ will be:

$$L = \begin{bmatrix}
1 & 1 & -3 & 0 & 1 \\
1 & 0 & -2 & 1 \\
0 & 0 & 0.5 & -0.5 & 1 \\
0 & 0 & 0.5 & -0.5 & 0 & 1
\end{bmatrix}, \quad D = \begin{bmatrix}
1 & 0 \\
2 & 0 \\
2 & 0
\end{bmatrix}$$

**Step 2** Form the $W$ matrix using the 2nd, 5th and 6th rows of the inverse of $L$:

$$W = \begin{bmatrix}
-1 & 1 & 0 & 0 & 0 \\
1 & 0 & 0.5 & 0.5 & 1 & 0 \\
1 & 0 & 0.5 & 0.5 & 0 & 1
\end{bmatrix}$$
**Step 3** Consider only the available boundary injections at buses 1, 2, 5, and 6. Form the candidate measurement Jacobian sub-matrix \( H_c \), given by:

\[
H_c = \begin{bmatrix}
2 & -1 & -1 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 \\
\end{bmatrix}
\]

where, the rows correspond to the candidate injection measurements at buses 1, 2, 5, and 6.

**Step 4** Form \( B = H_c W^T \):

\[
B = \begin{bmatrix}
-3 & 1.5 & 1.5 \\
3 & -1.5 & -1.5 \\
0 & 0.5 & -0.5 \\
0 & -0.5 & 0.5 \\
\end{bmatrix}
\]

and its reduced echelon form \( E \):

\[
E = \begin{bmatrix}
1 & 0 & -1 \\
0 & 0 & 0 \\
0 & 1 & -1 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

Hence, the injections at buses 1 and 5, corresponding to the linearly independent first and third rows of \( E \), should be placed in order to make the system fully observable.
Topological Observability Analysis

Measurement Assignment of Branches:

• If the branch flow is measured, the branch is assigned to its flow measurement.

• If an injection is measured at a terminal node of a branch, the branch will be assigned to that injection.

• Once a branch is assigned to a measurement, it cannot be assigned to any other measurement.

Observability as a Branch Coloring Problem:

Network observability problem can be defined as a branch coloring problem. The colors of the branches are determined by assuming distinct colors for the measurements and assigning measurements to the branches using measurement assignment rules. The network will be observable if a network tree can be formed such that none of the tree branches will have the same color.
Example 5:

![Graph showing measurements and colors assigned to branches]

### Measurement and Color Assignment Table

<table>
<thead>
<tr>
<th>Branch</th>
<th>Assigned Measurements</th>
<th>Measurements</th>
<th>Assigned Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>1,5</td>
<td>1</td>
<td>red</td>
</tr>
<tr>
<td>1-4</td>
<td>2</td>
<td>2</td>
<td>blue</td>
</tr>
<tr>
<td>1-3</td>
<td>6</td>
<td>3</td>
<td>green</td>
</tr>
<tr>
<td>2-4</td>
<td>5</td>
<td>4</td>
<td>yellow</td>
</tr>
<tr>
<td>3-5</td>
<td>6</td>
<td>5</td>
<td>black</td>
</tr>
<tr>
<td>4-6</td>
<td>4,7</td>
<td>6</td>
<td>brown</td>
</tr>
<tr>
<td>5-6</td>
<td>3,7</td>
<td>7</td>
<td>gray</td>
</tr>
</tbody>
</table>
Topological Observability Algorithm:

While the implementation of the topological observability analysis can be done in various ways, the essential steps of the algorithm can be stated in concise form as follows:

1. First assign all the flow measurements to their respective branches.

2. Injection measurements can be processed in an arbitrary sequence. The aim of introducing an injection measurement is to enlarge the existing forest by merging existing trees. If a new injection can not accomplish this, it will be declared as a redundant measurement.

Identifying the Observable Islands:

After processing all the flows and injections, if a spanning tree can not be found, then the observable islands need to be identified. This can be done as follows:

1. Discard those injections that have at least one incident branch which does not form a loop with the branches of the already defined forest.

2. Update the forest accordingly and repeat step 1 until no more injections need to be removed.
Example 6:

- Assign measurements to branches: $1 \Rightarrow 1-2$, $2 \Rightarrow 3-5$, $3 \Rightarrow 5-6$, $4 \Rightarrow 4-6$.

- Branches 1-4 and 2-4 are branches that are *not* forming loops with the branches of the existing forest, which contains branches 3-5, 5-6, and 1-2.
  Therefore, discard the injection at bus 4!

- The resulting observable islands are as follows:
Bad Data Detection and Identification

Chi-squares $\chi^2$ Test for Detecting Bad Data

Consider $X_1, X_2, \ldots X_N$, a set of $N$ independent random variables where:

$$X_i \sim N(0, 1)$$

Then, a new random variable $Y$ will have a $\chi^2$ distribution with $N$ degrees of freedom, i.e.:

$$\sum_{i=1}^{N} X_i^2 = Y \sim \chi^2_N$$

Now, consider the function $f(x)$:

$$f(x) = \sum_{i=1}^{m} R_{ii}^{-1} e_i^2 = \sum_{i=1}^{m} (\frac{e_i}{\sqrt{R_{ii}}})^2 = \sum_{i=1}^{m} (e_i^N)^2$$

and assuming:

$$e_i^N \sim N(0, 1)$$

$f(x)$ will have a $\chi^2$ distribution with at most $(m - n)$ degrees of freedom. In a power system, since at least $n$ measurements will have to satisfy the power balance equations, at most $(m - n)$ of the measurement errors will be linearly independent.
Bad Data Detection by $\chi^2$ Test

$Pr\{X \geq x_t\} = \int_{x_t}^{\infty} \chi^2(u) \cdot du$

Choose $x_t$ such that:

$Pr\{X \geq x_t\} = \alpha = 0.05$

Test:

If the measured $X \geq x_t$, then with 0.95 probability, bad data will be suspected.
The steps of the Chi-squares test can then be outlined as follows:

- Solve the WLS estimation problem and compute the objective function:

\[
J(\hat{x}) = \sum_{i=1}^{m} \frac{(z_i - h_i(\hat{x}))^2}{\sigma_i^2}
\]

where:
- \( \hat{x} \) : estimated state vector of dimension \( n \).
- \( h_i(\hat{x}) \) : estimated measurement \( i \).
- \( z_i \) : measured value of the measurement \( i \).
- \( \sigma_i^2 \) : variance of the measurement \( i \).
- \( m \) : number of measurements.

- Look up the value corresponding to \( p \) (e.g. 95 \%) probability and \((m - n)\) degrees of freedom, from the Chi-squares distribution table. Let this value be \( \chi^2_{(m-n),p} \).

Here \( p = Pr \left( J(\hat{x}) \leq \chi^2_{(m-n),p} \right) \).

- Test if \( J(\hat{x}) \geq \chi^2_{(m-n),p} \).

If yes, then bad data are detected.
Else, the measurements are not suspected to contain bad data.
Example 7:

\[ \begin{align*}
  x^T &= [ \theta_2, \theta_3, V_1, V_2, V_3 ] \\
  z^T &= [ P_{12}, Q_{12}, P_{13}, Q_{13}, P_2, Q_2, P_3, Q_3, V_1, V_3 ]
\end{align*} \]

No. of state variables, \( n = 5 \),
No. of measurements, \( m = 10 \),
The degrees of freedom, \( m - n = 10 - 5 = 5 \),
Probability of false alarm, \( (1 - p) = 0.05 \),

From the Chi-squares table \( \chi^2_{5,0.95} = 11.07 \).
In Matlab, use \( \text{chi2inv}(p,\text{df}) = \text{chi2inv}(0.95,5) = 11.07 \).

If the objective function \( J(\hat{x}) > 11.07 \), then
bad data will be suspected in the measurement set.
Properties of Measurement Residuals

Consider the state estimate in the linearized measurement model:

$$\Delta \hat{x} = (H^T R^{-1} H)^{-1} H^T R^{-1} \Delta z$$

and the estimated value of $\Delta z$:

$$\Delta \hat{z} = H \Delta \hat{x} = K \Delta z$$

where $K = H (H^T R^{-1} H)^{-1} H^T R^{-1}$ is called the hat matrix.

Now, the measurement residuals can be expressed as follows:

$$r = \Delta z - \Delta \hat{z}$$
$$= (I - K) \Delta z$$
$$= (I - K)(H \Delta x + e) \quad \text{[Note that } KH = H]\$$
$$= (I - K)e$$
$$= Se$$

where $S$ is called the residual sensitivity matrix.
Distribution of the Measurement Residuals

The residual covariance matrix $\Omega$ can be written as:

$$E[rr^T] = \Omega = S \cdot E[ee^T] \cdot S^T$$

$$= SRS^T$$

$$= SR$$

Hence, the normalized value of the residual for measurement $i$ will be given by:

$$r_i^N = \frac{r_i}{\sqrt{\Omega_{ii}}} = \frac{r_i}{\sqrt{R_{ii}S_{ii}}}.$$ 

$S$ is in general not symmetric.

Rank of $S$ is related to the measurement redundancy $(m - n)$ which is always smaller than its dimension $m$. Hence, it is a singular matrix.

$\Omega$ is always symmetric.
Measurement Classification

Measurements can be classified as critical and redundant (or non-critical) with the following properties:

- A critical measurement is the one whose elimination from the measurement set will result in an unobservable system. The row/column of $S$ corresponding to a critical measurement will be zero.

- The residuals of critical measurements will always be zero, and therefore errors in critical measurements can not be detected.

- Two measurements, none of which are critical, form a critical pair if removal of one causes the other one to become a critical measurement.

- Normalized residuals of a critical pair will be identical, hence bad data appearing in a critical pair can be detected but not identified.

- Errors in critical measurements will have no effect on the residuals of the remaining measurements. Hence, it is best to chose pseudo-measurements as critical measurements.
Largest Normalized Residual ($r^N$) Test

It can be shown that if there is a single bad data in the measurement set (provided that it is not a critical measurement) the largest normalized residual will correspond to bad data.

Steps of the largest normalized residual test for identification of single and non-interacting multiple bad data:

1. Solve the WLS estimation and obtain the elements of the measurement residual vector:

   $$r_i = z_i - h_i(\hat{x}), \quad i = 1, \ldots, m$$

2. Compute the normalized residuals:

   $$r^N_i = \frac{|r_i|}{\sqrt{\Omega_{ii}}} \quad i = 1, \ldots, m$$

3. Find $k$ such that $r^N_k$ is the largest among all $r^N_i$, $i = 1, \ldots, m$.

4. If $r^N_k > c$, then the $k$-th measurement will be suspected as bad data. Else, stop, no bad data will be suspected. Here, $c$ is a chosen identification threshold, e.g. 3.0.

5. Eliminate the $k$-th measurement from the measurement set and go to step 1.
Bad Data Identifiability Using the $r^N$ Test

**Single Bad Data**

- When there is single bad data, the largest normalized residual will correspond to the bad measurement, provided that it is not critical.

**Multiple Bad Data**

Multiple bad data may appear in 3 ways:

- Non-interacting:
  If $S_{ik} \approx 0$, then measurement $i$ and $k$ are said to be non-interacting. In this case, even if bad data appears simultaneously in both measurements, the largest normalized residual test can identify them sequentially, one pass at a time.

- Interacting, non-conforming:
  If $S_{ik}$ is significantly large, then measurements $i$ and $k$ are said to be interacting. However, if the errors in measurement $i$ and $k$ are not consistent with each other, then the largest normalized residual test may still indicate the bad data correctly.

- Interacting, conforming:
  If two interacting measurements have errors that are in agreement, then the largest normalized residual test may fail to identify either one.
In the figure, all branches have identical reactances, $x_i = j 0.1$, and all measurements have the same error variance $\sigma_i = 0.01$. Single, multiple interacting non-conforming and multiple interacting conforming bad data are introduced into the measurements and the results are given in the table below. Note that all measurements are assumed to be zero except for the bad data. The largest normalized residual for each case is typed in bold in the below table.

<table>
<thead>
<tr>
<th>Meas. Type</th>
<th>Bad Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Single</td>
</tr>
<tr>
<td></td>
<td>Non-conforming</td>
</tr>
<tr>
<td>$z_i$</td>
<td>$r_i^N$</td>
</tr>
<tr>
<td>Flow 1-3</td>
<td>0</td>
</tr>
<tr>
<td>Flow 2-1</td>
<td>0</td>
</tr>
<tr>
<td>Flow 3-2</td>
<td>1</td>
</tr>
<tr>
<td>Flow 2-3</td>
<td>0</td>
</tr>
<tr>
<td>Inj. 1</td>
<td>0</td>
</tr>
<tr>
<td>Inj. 3</td>
<td>0</td>
</tr>
</tbody>
</table>
Hypothesis Testing Identification (HTI)

- Method’s effectiveness depends upon the choice of an initial suspect measurement set which should include all bad data.
- Suspect set is chosen based on the normalized residuals.

Formulation

Then the sensitivity matrix $S$ and the error covariance matrix $R$ are partitioned according to the suspect and true measurements:

$$ r_s = S_{ss}e_s + S_{st}e_t $$
$$ r_t = S_{ts}e_s + S_{tt}e_t $$
$$ R = \begin{bmatrix} R_s & 0 \\ 0 & R_t \end{bmatrix} $$

Treating the last term of the first equation as noise, an estimate for $e_s$ can be obtained as:

$$ \hat{e}_s = S_{ss}^{-1}r_s $$

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**Statistical Properties of $\hat{e}_s$**

1. **Mean:**
   - If $E[e_t] = 0$, then $E[\hat{e}_s] = \hat{e}_s$.
   - Else, $E[\hat{e}_s] \neq \hat{e}_s$.

2. **Covariance:**
   - If $E[e_t] = 0$, then (after some algebra):
     \[
     Cov(e_s) = E[(e_s - \hat{e}_s)(e_s - \hat{e}_s)^T] = (S_{ss}^{-1} - I)R_s
     \]
   - Letting $T = S_{ss}^{-1}$,
     - if $e_{si}$ is in error: $\Rightarrow \hat{e}_{si} \sim N (e_{si}, \sigma_i^2(T_{ii} - 1))$
     - if $e_{si}$ is good: $\Rightarrow \hat{e}_{si} \sim N (0, \sigma_i^2 T_{ii})$
Hypotheses to be Tested:

$H_0$: measurement $i$ is valid.
$H_1$: measurement $i$ is in error.
Decision Rules:

A. Fixed probability of false alarm, $\alpha$

$\alpha = \Pr(\text{reject } H_0 \mid H_0 \text{ is true})$

$H_0$ is true $\implies \hat{e}_{si} \sim N(0, \sigma_i^2 T_{ii})$.

$\alpha = \Pr(\mid \hat{e}_{si} \mid > \lambda_i)$

Normalize:

$|\hat{e}_{si}| \implies \frac{|\hat{e}_{si}|}{\sigma_i \sqrt{T_{ii}}}$

Look up the cut-off value $N_{1-\alpha^2}$ from the Standard Normal distribution table. Then:

$\Pr \left( \frac{|\hat{e}_{si}|}{\sigma_i \sqrt{T_{ii}}} > N_{1-\alpha^2} \right) = \alpha$.

Therefore, $\lambda_i = \sigma_i \sqrt{T_{ii}} N_{(1-\alpha/2)}$
B. Fixed probability of bad data identification, \((1 - \beta)\)

\[
\beta = \Pr (\text{reject } H_1 \mid H_1 \text{ is true})
\]

\[
1 - \beta = \Pr (\text{reject } H_0 \mid H_1 \text{ is true})
\]

\[
= \Pr (\text{identification of bad data})
\]

\(H_1 \text{ is true} \implies \hat{e}_{si} \sim N (e_{si}, \sigma_i^2(T_{ii} - 1)).\)

\[
\beta = \Pr (\hat{e}_{si} \leq \lambda_i) - \Pr (\hat{e}_{si} \leq -\lambda_i)
\]

\[
\beta \approx \Pr (\hat{e}_{si} \leq \lambda_i)
\]

Normalize:

\[
\beta = \Pr \left( \frac{\hat{e}_{si} - |e_{si}|}{\sigma_i \sqrt{T_{ii} - 1}} \leq \frac{\lambda_i - |e_{si}|}{\sigma_i \sqrt{T_{ii} - 1}} \right)
\]

From part A \(\implies\) \(\lambda_i = \sigma_i \sqrt{T_{ii}} N_{(1-\frac{\alpha}{2})}\)

Substituting for \(\lambda_i, \ \sigma_i N_{\beta \sqrt{T_{ii}} - 1} = \sigma_i \sqrt{T_{ii}} N_{(1-\frac{\alpha}{2})- |e_{si}|}.\)
The steps of implementing the HTI method for fixed $\beta$ will now be outlined.

The following parameters should be fixed initially:

\[
a = \frac{|e_{s_i}|}{\sigma_i}, \quad (a = 40)
\]

\[
N_\beta = b, \quad (b = -2.32 \text{ for } \beta = 0.01)
\]

\[
N_{(1-\alpha/2)\text{max}} = 3.0
\]

**Steps of the Algorithm:**

1. Select suspect set $s_1$ based on $r^N$ and calculate

   \[
   T_{s_1} = S_{s_1,s_1}^{-1} \quad \text{and} \quad \hat{e}_{s_1} = T_{s_1}r_{s_1}
   \]

2. Calculate $N_{(1-\alpha/2)i}$:

   \[
   N_{(1-\alpha/2)i} = \frac{|e_{s_i}| + \sigma_i N_\beta \sqrt{T_{ii}} - 1}{\sigma_i \sqrt{T_{ii}}}
   \]

   with $0 \leq N_{(1-\alpha/2)i} \leq N_{(1-\alpha/2)\text{max}}$.

3. Calculate the threshold for each $s_{1i}$:

   \[
   \lambda_i = \sigma_i \sqrt{T_{ii}} N_{(1-\alpha/2)i}, \quad i = 1, \ldots, s_{1i}
   \]

4. Select measurement $s_{1i}$ if $|\hat{e}_{s_{1i}}| > \lambda_i$.

5. Form a shorter list of suspect measurements using those that are selected at step 4. Repeat steps 1-4 until all measurements that are suspected in the previous iteration are all selected again at step 4.
Exercise 4: [ Use the file exer04.pet for this exercise ]

This exercise is on network observability analysis.
Given the IEEE 14-bus system and its measurement configuration below, use ©P.E.T program to determine the observable islands and the unobservable branches.

Now, suggest locations and types of extra measurements to make the system fully observable. Note that the candidate measurements are:

- the injections at the boundary buses of observable islands, and
- the flow measurements on the branches connecting observable islands.

Place the suggested measurements and verify network observability by using the observability analysis function of the ©P.E.T program.
Exercise 5: [ Use the file exer05.pet for this exercise ]

This exercise is about bad data detection and identification tests. Both Chi-squares and largest normalized residual tests will be used to detect and identify bad data.

Consider the 3-bus system and its measurement configuration shown in the figure. The corresponding network and power flow data are given below:

<table>
<thead>
<tr>
<th>Line From Bus</th>
<th>Resistance (pu)</th>
<th>Reactance (pu)</th>
<th>Total Susceptance 2b (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2</td>
<td>0.01</td>
<td>0.03</td>
<td>0.0</td>
</tr>
<tr>
<td>1 3</td>
<td>0.02</td>
<td>0.05</td>
<td>0.0</td>
</tr>
<tr>
<td>2 3</td>
<td>0.03</td>
<td>0.08</td>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>P (MW)</th>
<th>Q (MVar)</th>
<th>P (MW)</th>
<th>Q (MVar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Slack)</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 (PQ)</td>
<td>0</td>
<td>0</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>3 (PQ)</td>
<td>0</td>
<td>0</td>
<td>50</td>
<td>40</td>
</tr>
</tbody>
</table>

Perform the following exercises using the ©P.E.T:

- Run the base case power flow. Generate measurements using the power flow results and run the WLS state estimation by choosing the Gaussian option for noise. Check for errors using the Chi-squares test. Note that, \( \chi^2_{0.95,5} = 11.1 \)

- Also check the normalized residuals by choosing Display Normalized Residuals option in the options/program settings/display settings menu. Use a detection threshold of 3.0 for the normalized residuals.

- Change the injection measurement \( P_2 \) value to -0.35 and repeat the above steps. Once you identify the bad data, remove it from the measurement set and rerun the WLS estimation until no more bad data are detected.
Exercise 6: [ Use the file exer06.pet for this exercise ]

This exercise illustrates the limitations of bad data processing methods. Some of the limitations are imposed by the measurement configuration and some are due to the method itself.

The network diagram and its associated measurement configuration of the IEEE 14-bus test system is shown below. Use the ©P.E.T program to answer the following questions:

1. Use the results of the normalized residual test to identify bad data for each case.

2. Explain the reason(s) of failure for those cases where the tests fail to detect and/or identify the bad data.

```
<table>
<thead>
<tr>
<th>Case No</th>
<th>Bad Measurement</th>
<th>Magnitude of Bad Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P Flow 6-13</td>
<td>+30</td>
</tr>
<tr>
<td>2</td>
<td>P Flow 9-14</td>
<td>+30</td>
</tr>
<tr>
<td>3</td>
<td>P Flow 4-9</td>
<td>+20</td>
</tr>
<tr>
<td></td>
<td>P Inj.at 4</td>
<td>+20</td>
</tr>
<tr>
<td>4</td>
<td>P Flow 3-4</td>
<td>+20</td>
</tr>
</tbody>
</table>
```

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